CHAPTER 6
Additional Topics in Trigonometry

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Section 6.1 Law of Sines

- If ABC is any oblique triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

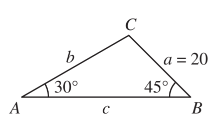
- You should be able to use the Law of Sines to solve an oblique triangle for the remaining three parts, given:
  (a) Two angles and any side (AAS or ASA)
  (b) Two sides and an angle opposite one of them (SSA)
  1. If A is acute and $h = b \sin A$:
     (a) $a < h$, no triangle is possible.
     (b) $a = h$ or $a > b$, one triangle is possible.
     (c) $h < a < b$, two triangles are possible.
  2. If A is obtuse and $h = b \sin A$:
     (a) $a \le b$, no triangle is possible.
     (b) $a > b$, one triangle is possible.
- The area of any triangle equals one-half the product of the lengths of two sides and the sine of their included angle.

$$A = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}bc \sin A$$

Vocabulary Check

1. oblique **2.** $\dfrac{b}{\sin B}$ **3.** $\dfrac{1}{2} ac \sin B$

1.



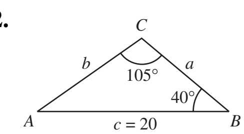
Given: $A = 30°$, $B = 45°$, $a = 20$

$C = 180° - A - B = 105°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{20 \sin 45°}{\sin 30°} = 20\sqrt{2} \approx 28.28$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{20 \sin 105°}{\sin 30°} \approx 38.64$$

2.



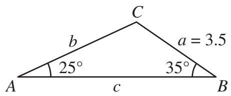
Given: $B = 40°$, $C = 105°$, $c = 20$

$A = 180° - B - C = 35°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{20 \sin 35°}{\sin 105°} \approx 11.88$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{20 \sin 40°}{\sin 105°} \approx 13.31$$

3.



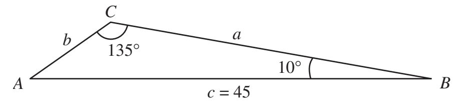
Given: $A = 25°, B = 35°, a = 3.5$

$C = 180° - A - B = 120°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{3.5}{\sin 25°}(\sin 35°) \approx 4.75$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{3.5}{\sin 25°}(\sin 120°) \approx 7.17$$

4.



Given: $B = 10°, C = 135°, c = 45$

$A = 180° - B - C = 35°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{45 \sin 35°}{\sin 135°} \approx 36.50$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{45 \sin 10°}{\sin 135°} \approx 11.05$$

5. Given: $A = 36°,\ a = 8,\ b = 5$

$$\sin B = \frac{b \sin A}{a} = \frac{5 \sin 36°}{8} \approx 0.36737 \implies B \approx 21.55°$$

$$C = 180° - A - B \approx 180° - 36° - 21.55 = 122.45°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{8}{\sin 36°}(\sin 122.45°) \approx 11.49$$

6. Given: $A = 60°, a = 9, c = 10$

$$\sin C = \frac{c \sin A}{a} = \frac{10 \sin 60°}{9} \approx 0.9623 \implies C \approx 74.21° \text{ or } C \approx 105.79°$$

Case 1

$C \approx 74.21°$

$B = 180° - A - C \approx 45.79°$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 45.79°}{\sin 60°} \approx 7.45$$

Case 2

$C \approx 105.79°$

$B = 180° - A - C \approx 14.21°$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 14.21°}{\sin 60°} \approx 2.55$$

7. Given: $A = 102.4°, C = 16.7°, a = 21.6$

$B = 180° - A - C = 60.9°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{21.6}{\sin 102.4°}(\sin 60.9°) \approx 19.32$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{21.6}{\sin 102.4°}(\sin 16.7°) \approx 6.36$$

8. Given: $A = 24.3°, C = 54.6°, c = 2.68$

$B = 180° - A - C = 101.1°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{2.68 \sin 24.3°}{\sin 54.6°} \approx 1.35$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{2.68 \sin 101.1°}{\sin 54.6°} \approx 3.23$$

9. Given: $A = 83° 20',\ C = 54.6°,\ c = 18.1$

$$B = 180° - A - C = 180° - 83° 20' - 54° 36' = 42° 4'$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{18.1}{\sin 54.6°}(\sin 83° 20') \approx 22.05$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{18.1}{\sin 54.6°}(\sin 42° 4') \approx 14.88$$

10. Given: $A = 5° 40', B = 8° 15', b = 4.8$

$C = 180° - A - B = 166° 5'$

$$a = \frac{b}{\sin B}(\sin A) = \frac{4.8 \sin 5° 40'}{\sin 8° 15'} \approx 3.30$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{4.8 \sin 166° 5'}{\sin 8° 15'} \approx 8.05$$

11. Given: $B = 15° \, 30'$, $a = 4.5$, $b = 6.8$

$$\sin A = \frac{a \sin B}{b} = \frac{4.5 \sin 15° \, 30'}{6.8} \approx 0.17685 \implies A \approx 10° \, 11'$$

$$C = 180° - A - B \approx 180° - 10° \, 11' - 15° \, 30' = 154° \, 19'$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{6.8}{\sin 15° \, 30'}(\sin 154° \, 19') \approx 11.03$$

12. Given: $B = 2° \, 45'$, $b = 6.2$, $c = 5.8$

$$\sin C = \frac{c \sin B}{b} = \frac{5.8 \sin 2° \, 45'}{6.2} \approx 0.04488 \implies C \approx 2.57° \text{ or } 2° \, 34'$$

$$A = 180 - B - C \approx 174.68°, \text{ or } 174° \, 41'$$

$$a = \frac{b}{\sin B}(\sin A) \approx \frac{6.2 \sin 174.68°}{\sin 2° \, 45'} \approx 11.99$$

13. Given: $C = 145°$, $b = 4$, $c = 14$

$$\sin B = \frac{b \sin C}{c} = \frac{4 \sin 145°}{14} \approx 0.16388 \implies B \approx 9.43°$$

$$A = 180° - B - C \approx 180° - 9.43° - 145° = 25.57°$$

$$a = \frac{c}{\sin C}(\sin A) \approx \frac{14}{\sin 145°}(\sin 25.57°) \approx 10.53$$

14. Given: $A = 100°$, $a = 125$, $c = 10$

$$\sin C = \frac{c \sin A}{a} = \frac{10 \sin 100°}{125} \approx 0.07878 \implies C \approx 4.52°$$

$$B = 180° - A - C \approx 75.48°$$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{125 \sin 75.48°}{\sin 100°} \approx 122.87$$

15. Given: $A = 110° \, 15'$, $a = 48$, $b = 16$

$$\sin B = \frac{b \sin A}{a} = \frac{16 \sin 110° \, 15'}{48} \approx 0.31273 \implies B \approx 18° \, 13'$$

$$C = 180° - A - B \approx 180° - 110° \, 15' - 18° \, 13' = 51° \, 32'$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{48}{\sin 110° \, 15'}(\sin 51° \, 32') \approx 40.06$$

16. Given: $C = 85° \, 20'$, $a = 35$, $c = 50$

$$\sin A = \frac{a \sin C}{c} = \frac{35 \sin 85° \, 20'}{50} \approx 0.6977 \implies A \approx 44.24°, \text{ or } 44° \, 14'$$

$$B = 180° - A - C \approx 50.43°, \text{ or } 50° \, 26'$$

$$b = \frac{C \sin B}{\sin C} \approx \frac{50 \sin 50.43°}{\sin 85° \, 20'} \approx 38.67$$

17. Given: $A = 55°$, $B = 42°$, $c = \frac{3}{4}$

$$C = 180° - A - B = 83°$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{0.75}{\sin 83°}(\sin 55°) \approx 0.62$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{0.75}{\sin 83°}(\sin 42°) \approx 0.51$$

18. Given: $B = 28°$, $C = 104°$, $a = 3\frac{5}{8}$

$$A = 180° - B - C = 48°$$

$$b = \frac{a \sin B}{\sin A} = \frac{3\frac{5}{8} \sin 28°}{\sin 48°} \approx 2.29$$

$$c = \frac{a \sin C}{\sin A} = \frac{3\frac{5}{8} \sin 104°}{\sin 48°} \approx 4.73$$

19. Given: $A = 110°, a = 125, b = 100$

$\sin B = \dfrac{b \sin A}{a} = \dfrac{100 \sin 110°}{125} \approx 0.75175 \implies B \approx 48.74°$

$C = 180° - A - B \approx 21.26°$

$c = \dfrac{a \sin C}{\sin A} \approx \dfrac{125 \sin 21.26°}{\sin 110°} \approx 48.23$

20. Given: $a = 125,\ b = 200,\ A = 110°$

No triangle is formed because A is obtuse and $a < b$.

21. Given: $a = 18, b = 20, A = 76°$

$h = 20 \sin 76° \approx 19.41$

Since $a < h$, no triangle is formed.

22. Given: $A = 76°, a = 34, b = 21$

$\sin B = \dfrac{b \sin A}{a} = \dfrac{21 \sin 76°}{34} \approx 0.5993 \implies B \approx 36.82°$

$C = 180° - A - B \approx 67.18°$

$c = \dfrac{a \sin C}{\sin A} \approx \dfrac{34 \sin 67.18°}{\sin 76°} \approx 32.30$

23. Given: $A = 58°, a = 11.4, c = 12.8$

$\sin B = \dfrac{b \sin A}{a} = \dfrac{12.8 \sin 58°}{11.4} \approx 0.9522 \implies B \approx 72.21°$ or $B \approx 107.79°$

<u>Case 1</u>

$B \approx 72.21°$

$C = 180° - A - B \approx 49.79°$

$c = \dfrac{a}{\sin A}(\sin C) \approx \dfrac{11.4 \sin 49.79°}{\sin 58°} \approx 10.27$

<u>Case 2</u>

$B \approx 107.79°$

$C = 180° - A - B \approx 14.21°$

$c = \dfrac{a}{\sin A}(\sin C) \approx \dfrac{11.4 \sin 14.21°}{\sin 58°} \approx 3.30$

24. Given: $a = 4.5,\ b = 12.8,\ A = 58°$

$h = 12.8 \sin 58° \approx 10.86$

Since $a < h$, no triangle is formed.

25. Given: $A = 36°,\ a = 5$

(a) One solution if $b \le 5$ or $b = \dfrac{5}{\sin 36°}$

(b) Two solutions if $5 < b < \dfrac{5}{\sin 36°}$

(c) No solution if $b > \dfrac{5}{\sin 36°}$

26. Given: $A = 60°, a = 10$

(a) One solution if $b \le 10$ or $b = \dfrac{10}{\sin 60°}$.

(b) Two solutions if $10 < b < \dfrac{10}{\sin 60°}$.

(c) No solutions if $b > \dfrac{10}{\sin 60°}$.

27. Given: $A = 10°, a = 10.8$

(a) One solution if $b \le 10.8$ or $b = \dfrac{10.8}{\sin 10°}$

(b) Two solutions if $10.8 < b < \dfrac{10.8}{\sin 10°}$

(c) No solution if $b > \dfrac{10.8}{\sin 10°}$

28. Given: $A = 88°, a = 315.6$

(a) One solution if $b \le 315.6$ or $b = \dfrac{315.6}{\sin 88°}$

(b) Two solutions if $315.6 < b < \dfrac{315.6}{\sin 88°}$

(c) No solutions if $b > \dfrac{315.6}{\sin 88°}$

29. Area $= \dfrac{1}{2}ab \sin C = \dfrac{1}{2}(4)(6) \sin 120° \approx 10.4$

30. Area $= \frac{1}{2}ac \sin B = \frac{1}{2}(62)(20) \sin 130° \approx 474.9$

31. Area $= \frac{1}{2}bc \sin A = \frac{1}{2}(57)(85) \sin 43° 45' \approx 1675.2$

32. $A = 5° 15', b = 4.5, c = 22$

Area $= \frac{1}{2}bc \sin A$

$= \left(\frac{1}{2}\right)(4.5)(22) \sin 5.25° \approx 4.5$

33. Area $= \frac{1}{2}ac \sin B = \frac{1}{2}(105)(64)\sin(72°30') \approx 3204.5$

34. $C = 84° 30', a = 16, b = 20$

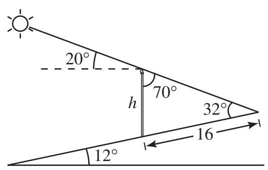
Area $= \dfrac{1}{2}ab \sin C$

$= \left(\dfrac{1}{2}\right)(16)(20) \sin 84.5° \approx 159.3$

35. $C = 180° - 23° - 94° = 63°$

$h = \dfrac{35}{\sin 63°}(\sin 23°) \approx 15.3$ meters

36. (a)



(b) $\dfrac{h}{\sin 32°} = \dfrac{16}{\sin 70°}$

(c) $h = \dfrac{16 \sin 32°}{\sin 70°} \approx 9.0$ meters

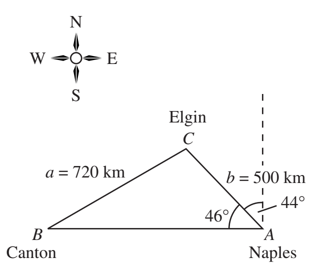
37. $\dfrac{\sin(42° - \theta)}{10} = \dfrac{\sin 48°}{17}$

$\sin(42° - \theta) \approx 0.43714$

$42° - \theta \approx 25.9°$

$\theta \approx 16.1°$

38.



Given: $A = 46°, a = 720, b = 500$

$\sin B = \dfrac{b \sin A}{a} = \dfrac{500 \sin 46°}{720} \approx 0.50 \implies B \approx 30°$

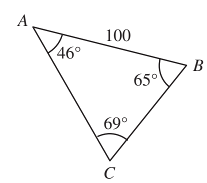
The bearing from C to B is 240°.

39. Given: $c = 100$

$A = 74° - 28° = 46°,$

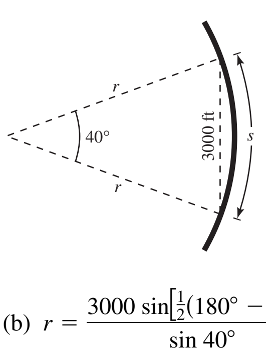
$B = 180° - 41° - 74° = 65°,$

$C = 180° - 46° - 65° = 69°$



$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{100}{\sin 69°}(\sin 46°) \approx 77$ meters

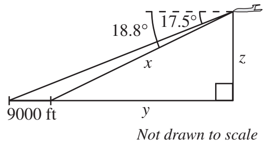
40.



(b) $r = \dfrac{3000 \sin\left[\frac{1}{2}(180° - 40°)\right]}{\sin 40°} \approx 4385.71$ feet

(c) $s \approx 40\left(\dfrac{\pi}{180}\right)4385.71 \approx 3061.80$ feet

41. (a)



(b) $\dfrac{x}{\sin 17.5°} = \dfrac{9000}{\sin 1.3°}$

$x \approx 119{,}289.1261$ feet ≈ 22.6 miles

(c) $\dfrac{y}{\sin 71.2°} = \dfrac{x}{\sin 90°}$

$y = x \sin 71.2° \approx 119{,}289.1261 \sin 71.2°$

$\approx 112{,}924.963$ feet ≈ 21.4 miles

(d) $z = x \sin 18.8° \approx 119{,}289.1261 \sin 18.8°$

$\approx 38{,}443$ feet ≈ 7.3 miles

42. Given: $A = 15°, B = 135°, c = 30$

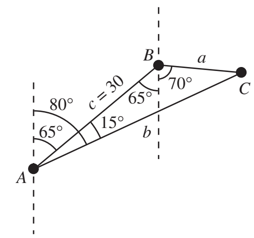
$C = 180° - A - B = 30°$

From Pine Knob:

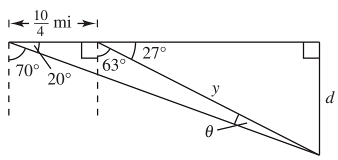
$b = \dfrac{c \sin B}{\sin C} = \dfrac{30 \sin 135°}{\sin 30°} \approx 42.4$ kilometers

From Colt Station:

$a = \dfrac{c \sin A}{\sin C} = \dfrac{30 \sin 15°}{\sin 30°} \approx 15.5$ kilometers



43.



In 15 minutes the boat has traveled

$(10 \text{ mph})\left(\frac{1}{4} \text{ hr}\right) = \frac{10}{4}$ miles.

$\theta = 180° - 20° - (90° + 63°)$

$\theta = 7°$

$\dfrac{10/4}{\sin 7°} = \dfrac{y}{\sin 20°}$

$y \approx 7.0161$

$\sin 27° = \dfrac{d}{7.0161}$

$d \approx 3.2$ miles

44. (a) $\sin \alpha = \dfrac{5.45}{58.36} \approx 0.0934 \implies \alpha \approx 5.36°$

(b) $\dfrac{\sin \beta}{d} = \dfrac{\sin \theta}{58.36}$

$\sin \beta = \dfrac{d \sin \theta}{58.36}$

$\beta = \sin^{-1}\left(\dfrac{d \sin \theta}{58.36}\right)$

(c) $\dfrac{d}{\sin(84.64° - \theta)} = \dfrac{58.36}{\sin \theta}$ or

$d = \dfrac{58.36 \sin(84.64° - \theta)}{\sin \theta}$

(d)

θ	10°	20°	30°	40°	50°	60°
d	324.1	154.2	95.19	63.80	43.30	28.10

45. True. If one angle of a triangle is obtuse, then there is less than 90° left for the other two angles, so it cannot contain a right angle. It must be oblique.

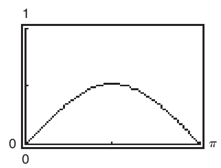
46. False. Two sides and one opposite angle do not necessarily determine a unique triangle.

47. (a) $\dfrac{\sin\alpha}{9} = \dfrac{\sin\beta}{18}$

$\sin\alpha = 0.5\sin\beta$

$\alpha = \arcsin(0.5\sin\beta)$

(b)



Domain: $0 < \beta < \pi$

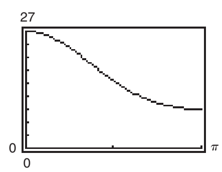
Range: $0 < \alpha \le \dfrac{\pi}{6}$

(c) $\gamma = \pi - \alpha - \beta = \pi - \beta - \arcsin(0.5\sin\beta)$

$\dfrac{c}{\sin\gamma} = \dfrac{18}{\sin\beta}$

$c = \dfrac{18\sin\gamma}{\sin\beta} = \dfrac{18\sin[\pi - \beta - \arcsin(0.5\sin\beta)]}{\sin\beta}$

(d)



Domain: $0 < \beta < \pi$

Range: $9 < c < 27$

(e)

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α	0.1960	0.3669	0.4848	0.5234	0.4720	0.3445	0.1683
c	25.95	23.07	19.19	15.33	12.29	10.31	9.27

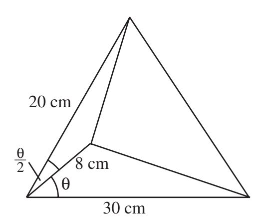
As $\beta \longrightarrow 0,\ c \longrightarrow 27$

As $\beta \longrightarrow \pi,\ c \longrightarrow 9$

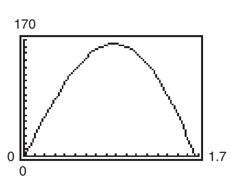
48. (a) $A = \dfrac{1}{2}(30)(20)\sin\left(\theta + \dfrac{\theta}{2}\right) - \dfrac{1}{2}(8)(20)\sin\dfrac{\theta}{2} - \dfrac{1}{2}(8)(30)\sin\theta$

$= 300\sin\dfrac{3\theta}{2} - 80\sin\dfrac{\theta}{2} - 120\sin\theta$

$= 20\left[15\sin\dfrac{3\theta}{2} - 4\sin\dfrac{\theta}{2} - 6\sin\theta\right]$



(b)



(c) Domain: $0 \le \theta \le 1.6690$

The domain would increase in length and the area would have a greater maximum value if the 8-centimeter line segment were decreased.

49. $\sin x\cot x = \sin x\dfrac{\cos x}{\sin x} = \cos x$

50. $\tan x\cos x\sec x = \tan x\cos x\dfrac{1}{\cos x} = \tan x$

51. $1 - \sin^2\left(\dfrac{\pi}{2} - x\right) = 1 - \cos^2 x = \sin^2 x$

52. $1 + \cot^2\left(\dfrac{\pi}{2} - x\right) = 1 + \tan^2 x = \sec^2 x$

53. $6\sin 8\theta\cos 3\theta = (6)\left(\frac{1}{2}\right)[\sin(8\theta + 3\theta) + \sin(8\theta - 3\theta)]$

$= 3(\sin 11\theta + \sin 5\theta)$

54. $2\cos 5\theta\sin 2\theta = 2\cdot\frac{1}{2}[\sin(5\theta + 2\theta) - \sin(5\theta - 2\theta)]$

$= \sin 7\theta - \sin 3\theta$

Section 6.2 Law of Cosines

- If ABC is any oblique triangle with sides a, b, and c, the following equations are valid.

  (a) $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

  (b) $b^2 = a^2 + c^2 - 2ac\cos B$ or $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

  (c) $c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

- You should be able to use the Law of Cosines to solve an oblique triangle for the remaining three parts, given:

  (a) Three sides (SSS)

  (b) Two sides and their included angle (SAS)

- Given any triangle with sides of length a, b, and c, the area of the triangle is

  $$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}. \qquad \text{(Heron's Formula)}$$

Vocabulary Check

1. Cosines

2. $b^2 = a^2 + c^2 - 2ac\cos B$

3. Heron's Area

1. Given: $a = 7, b = 10, c = 15$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{49 + 100 - 225}{2(7)(10)} \approx -0.5429 \implies C \approx 122.88^\circ$$

$$\sin B = \frac{b\sin C}{c} = \frac{10\sin 122.88^\circ}{15} \approx 0.5599 \implies B \approx 34.05^\circ$$

$$A \approx 180^\circ - 34.05^\circ - 122.88^\circ \approx 23.07^\circ$$

2. Given: $a = 8, b = 3, c = 9$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{8^2 + 3^2 - 9^2}{2(8)(3)} \approx -0.16667 \implies C = 99.59^\circ$$

$$\sin A = \frac{a\sin C}{c} = \frac{8\sin 99.59^\circ}{9} = 0.8765 \implies A = 61.22^\circ$$

$$B \approx 180^\circ - 61.22^\circ - 99.59^\circ \approx 19.19^\circ$$

3. Given: $A = 30^\circ, \; b = 15, \; c = 30$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 225 + 900 - 2(15)(30)\cos 30^\circ \approx 345.5771$$

$$a \approx 18.59$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{(18.59)^2 + 15^2 - 30^2}{2(18.59)(15)} \approx -0.5907 \implies C \approx 126.21^\circ$$

$$B \approx 180^\circ - 30^\circ - 126.21^\circ = 13.79^\circ$$

4. Given: $C = 105°, a = 10, b = 4.5$

$$c^2 = a^2 + b^2 - 2ab\cos C = 10^2 + 4.5^2 - 2(10)(4.5)\cos 105° \approx 143.5437 \implies c \approx 11.98$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{10^2 + (12.0)^2 - (4.5)^2}{2(10)(12.0)} \approx 0.93187 \implies B \approx 21.27°$$

$$A = 180° - 105° - 21.27° \approx 53.73°$$

5. $a = 11, b = 14, c = 20$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{121 + 196 - 400}{2(11)(14)} \approx -0.2695 \implies C \approx 105.63°$$

$$\sin B = \frac{b\sin C}{c} = \frac{14\sin 105.63°}{20} \approx 0.6741 \implies B \approx 42.38°$$

$$A \approx 180° - 42.38° - 105.63° \approx 31.99°$$

6. Given: $a = 55, b = 25, c = 72$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{55^2 + 25^2 - 72^2}{2(55)(25)} \approx -0.5578 \implies C \approx 123.91°$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25^2 + 72^2 - 55^2}{2(25)(72)} \approx 0.7733 \implies A \approx 39.35°$$

$$B = 180° - 123.91° - 39.35° \approx 16.74°$$

7. Given: $a = 75.4, \ b = 52, \ c = 52$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{52^2 + 52^2 - 75.4^2}{2(52)(52)} = -0.05125 \implies A \approx 92.94°$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{52(0.9987)}{75.4} \approx 0.68876 \implies B \approx 43.53°$$

$$C = B \approx 43.53°$$

8. Given: $a = 1.42, b = 0.75, c = 1.25$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(0.75)^2 + (1.25)^2 - (1.42)^2}{2(0.75)(1.25)} = 0.05792 \implies A \approx 86.68°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(1.42)^2 + (1.25)^2 - (0.75)^2}{2(1.42)(1.25)} \approx 0.84969 \implies B \approx 31.82°$$

$$C = 180° - 86.68° - 31.82° \approx 61.50°$$

9. Given: $A = 135°, b = 4, c = 9$

$$a^2 = b^2 + c^2 - 2bc\cos A = 16 + 81 - 2(4)(9)\cos 135° \approx 147.9117 \implies a \approx 12.16$$

$$\sin B = \frac{b\sin A}{a} = \frac{4\sin 135°}{12.16} \approx 0.2326 \implies B \approx 13.45°$$

$$C \approx 180° - 135° - 13.45° \approx 31.55°$$

10. Given: $A = 55°, b = 3, c = 10$

$$a^2 = b^2 + c^2 - 2bc\cos A = 3^2 + 10^2 - 2(3)(10)\cos 55° \approx 74.585 \implies a \approx 8.64$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{3\sin 55°}{8.636} \approx 0.2846 \implies A \approx 16.53°$$

$$C \approx 180° - 16.53° - 55° \approx 108.47°$$

11. Given: $B = 10° 35'$, $a = 40$, $c = 30$

$b^2 = a^2 + c^2 - 2ac \cos B = 1600 + 900 - 2(40)(30)\cos 10° 35' \approx 140.8268 \implies b \approx 11.87$

$\sin C = \dfrac{c \sin B}{b} = \dfrac{30 \sin 10° 35'}{11.87} \approx 0.4642 \implies C \approx 27.66° \approx 27° 40'$

$A \approx 180° - 10° 35' - 27° 40' = 141° 45'$

12. Given: $B = 75° 20'$, $a = 6.2$, $c = 9.5$

$b^2 = a^2 + c^2 - 2ac \cos B = (6.2)^2 + (9.5)^2 - 2(6.2)(9.5) \cos 75° 20' \approx 98.8636 \implies b \approx 9.94$

$\sin A = \dfrac{a \sin B}{b} \approx \dfrac{6.2 \sin 75° 20'}{9.94} \approx 0.6034 \implies A \approx 37.1°$, or $37° 6'$

$C \approx 180° - 75° 20' - 37° 6' \approx 67° 34'$

13. Given: $B = 125° 40'$, $a = 32$, $c = 32$

$b^2 = a^2 + c^2 - 2ac \cos B \approx 32^2 + 32^2 - 2(32)(32) \cos 125° 40' \approx 3242.1888 \implies b \approx 56.94$

$A = C \implies 2A = 180° - 125° 40' = 54° 20' \implies A = C = 27° 10'$

14. Given: $C = 15° 15'$, $a = 6.25$, $b = 2.15$

$c^2 = a^2 + b^2 - 2ab \cos C = (6.25)^2 + (2.15)^2 - 2(6.25)(2.15) \cos 15° 15' \approx 17.7563 \implies c \approx 4.21$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \approx \dfrac{(2.15)^2 + (4.2138)^2 - (6.25)^2}{2(2.15)(4.2138)} \approx -0.9208 \implies A \approx 157.04°$ or $157° 2'$

$B \approx 180° - 15° 15' - 157.04° \approx 7.7°$ or $7° 43'$

15. $C = 43°$, $a = \dfrac{4}{9}$, $b = \dfrac{7}{9}$

$c^2 = a^2 + b^2 - 2ab \cos C = \left(\dfrac{4}{9}\right)^2 + \left(\dfrac{7}{9}\right)^2 - 2\left(\dfrac{4}{9}\right)\left(\dfrac{7}{9}\right)\cos 43° \approx 0.2968 \implies c \approx 0.54$

$\sin A = \dfrac{a \sin C}{c} = \dfrac{(4/9) \sin 43°}{0.5448} \approx 0.5564 \implies A \approx 33.80°$

$B \approx 180° - 43° - 33.8° \approx 103.20°$

16. Given: $C = 103°$, $a = \dfrac{3}{8}$, $b = \dfrac{3}{4}$

$c^2 = a^2 + b^2 - 2ab \cos C = \left(\dfrac{3}{8}\right)^2 + \left(\dfrac{3}{4}\right)^2 - 2\left(\dfrac{3}{8}\right)\left(\dfrac{3}{4}\right) \cos 103° \approx 0.8297 \implies c \approx 0.91$

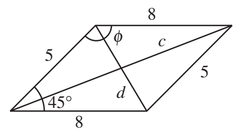
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \approx \dfrac{\left(\dfrac{3}{4}\right)^2 + (0.91)^2 - \left(\dfrac{3}{8}\right)^2}{2\left(\dfrac{3}{4}\right)(0.91)} \approx 0.9160 \implies A \approx 23.65°$

$B \approx 180° - 23.65° - 103° \approx 53.35°$

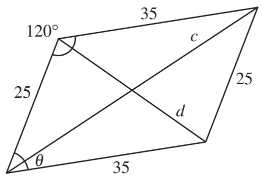
17. $d^2 = 5^2 + 8^2 - 2(5)(8)\cos 45° \approx 32.4315 \implies d \approx 5.69$

$2\phi = 360° - 2(45°) = 270° \implies \phi = 135°$

$c^2 = 5^2 + 8^2 - 2(5)(8)\cos 135° \approx 145.5685 \implies c \approx 12.07$

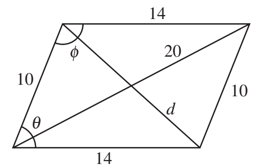


18.



$$c^2 = 25^2 + 35^2 - 2(25)(35)\cos 120^\circ$$
$$= 2725 \implies c \approx 52.20$$
$$\theta = \frac{1}{2}[360^\circ - 2(120^\circ)] = 60^\circ$$
$$d^2 = 25^2 + 35^2 - 2(25)(35)\cos 60^\circ$$
$$= 975 \implies d \approx 31.22$$

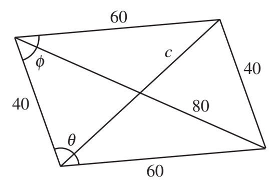
19.



$$\cos\phi = \frac{10^2 + 14^2 - 20^2}{2(10)(14)}$$
$$\phi \approx 111.8^\circ$$
$$2\theta \approx 360^\circ - 2(111.8^\circ)$$
$$\theta = 68.2^\circ$$
$$d^2 = 10^2 + 14^2 - 2(10)(14)\cos 68.2^\circ$$
$$d \approx 13.86$$

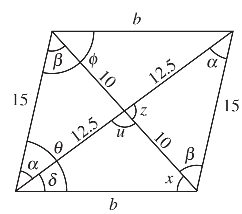
20. $\cos\theta = \dfrac{40^2 + 60^2 - 80^2}{2(40)(60)} = -\dfrac{1}{4} \implies \theta \approx 104.5^\circ$

$$\phi \approx \frac{1}{2}[360^\circ - 2(104.5^\circ)] \approx 75.5^\circ$$
$$c^2 \approx 40^2 + 60^2 - 2(40)(60)\cos 75.5^\circ = 3998$$
$$c \approx 63.23$$



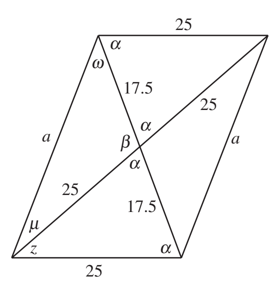
21. $\cos\alpha = \dfrac{(12.5)^2 + (15)^2 - 10^2}{2(12.5)(15)} = 0.75 \implies \alpha \approx 41.41^\circ$

$$\cos\beta = \frac{10^2 + 15^2 - (12.5)^2}{2(10)(15)} = 0.5625 \implies \beta \approx 55.77^\circ$$
$$z = 180^\circ - \alpha - \beta = 82.82^\circ$$
$$u = 180^\circ - z = 97.18^\circ$$
$$b^2 = 12.5^2 + 10^2 - 2(12.5)(10)\cos 97.18^\circ \approx 287.4967 \implies b \approx 16.96$$
$$\cos\delta = \frac{12.5^2 + 16.96^2 - 10^2}{2(12.5)(16.96)} \approx 0.8111 \implies \delta \approx 35.80^\circ$$
$$\theta = \alpha + \delta = 41.41^\circ + 35.80^\circ \approx 77.2^\circ$$
$$2\phi = 360^\circ - 2\theta \implies \phi = \frac{360^\circ - 2(77.21^\circ)}{2} = 102.8^\circ$$



22. $\cos\alpha = \dfrac{25^2 + 17.5^2 - 25^2}{2(25)(17.5)}$

$$\alpha \approx 69.512^\circ$$
$$\beta \approx 180 - \alpha \approx 110.488^\circ$$
$$a^2 = 17.5^2 + 25^2 - 2(17.5)(25)\cos 110.488^\circ$$
$$a \approx 35.18$$
$$z = 180 - 2\alpha \approx 40.975^\circ$$
$$\cos\mu = \frac{25^2 + 35.18^2 - 17.5^2}{2(25)(35.18)}$$
$$\mu \approx 27.775^\circ$$
$$\theta = \mu + z \approx 68.7^\circ$$
$$\omega = 180^\circ - \mu - \beta \approx 41.738^\circ$$
$$\phi = \omega + \alpha \approx 111.3^\circ$$



23. $a = 5,\ b = 7,\ c = 10 \implies s = \dfrac{a+b+c}{2} = 11$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{11(6)(4)(1)} \approx 16.25$

24. $a = 12,\ b = 15,\ c = 9 \implies s = \dfrac{12+15+9}{2} = 18$

Area $= \sqrt{18(6)(3)(9)} = 54$

25. $a = 2.5, b = 10.2, c = 9 \implies s = \dfrac{a+b+c}{2} = 10.85$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10.85(8.35)(0.65)(1.85)} \approx 10.4$

26. Given: $a = 75.4, b = 52, c = 52$

$s = \dfrac{75.4 + 52 + 52}{2} = 89.7$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{89.7(14.3)(37.7)(37.7)} \approx 1350.2$

27. $a = 12.32, b = 8.46, c = 15.05 \implies s = \dfrac{a+b+c}{2} = 17.915$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{17.915(5.595)(9.455)(2.865)} \approx 52.11$

28. Given: $a = 3.05, b = 0.75, c = 2.45$

$s = \dfrac{3.05 + 0.75 + 2.45}{2} = 3.125$

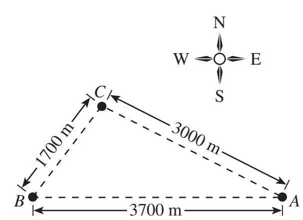
Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{3.125(0.075)(2.375)(0.675)} \approx 0.61$

29. $\cos B = \dfrac{1700^2 + 3700^2 - 3000^2}{2(1700)(3700)} \implies B \approx 52.9°$

Bearing: $90° - 52.9° =$ N 37.1° E

$\cos C = \dfrac{1700^2 + 3000^2 - 3700^2}{2(1700)(3000)} \implies C \approx 100.2°$

Bearing: $90° - 26.9° =$ S 63.1° E



30. Distance from Franklin to Rosemount:

$d = \sqrt{810^2 + 648^2 - 2(810)(648)\cos(137°)}$

≈ 1357.8 miles

Bearing from Franklin to Rosemount:

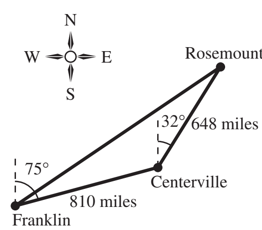
N $(75° - \theta)$ E

$\cos\theta \approx \dfrac{(1357.8)^2 + 810^2 - 648^2}{2(1357.8)(810)}$

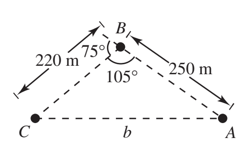
≈ 0.9456

$\theta \approx 19.0°$

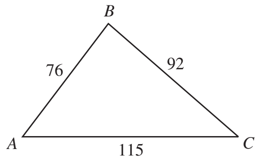
Bearing from Franklin to Rosemont: N 56.0° E



31. $b^2 = 220^2 + 250^2 - 2(220)(250)\cos 105° \implies b \approx 373.3$ meters



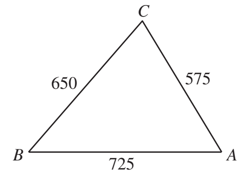
32.



$$\cos A = \frac{115^2 + 76^2 - 92^2}{2(115)(76)} \approx 0.6028 \implies A \approx 52.9^\circ$$

$$\cos C = \frac{115^2 + 92^2 - 76^2}{2(115)(92)} \approx 0.75203 \implies c \approx 41.2^\circ$$

33.



The largest angle is across from the largest side.

$$\cos C = \frac{650^2 + 575^2 - 725^2}{2(650)(575)}$$

$$C \approx 72.3^\circ$$

34. $\cos\theta = \dfrac{2^2 + 3^2 - (4.5)^2}{2(2)(3)} \approx -0.60417$

$\theta \approx 127.2^\circ$

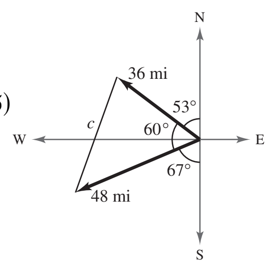
35. $C = 180^\circ - 53^\circ - 67^\circ = 60^\circ$

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 36^2 + 48^2 - 2(36)(48)(0.5)$

$= 1872$

$c \approx 43.3$ mi



36. The angles at the base of the tower are 96° and 84°. The longer guy wire g_1 is given by:

$g_1{}^2 = 75^2 + 100^2 - 2(75)(100)\cos 96^\circ$

$\approx 17{,}192.9 \implies g_1 \approx 131.1$ feet

The shorter guy wire g_2 is given by:

$g_2{}^2 = 75^2 + 100^2 - 2(75)(100)\cos 84$

$\approx 14{,}057.1 \implies g_2 \approx 118.6$ feet

37. (a) $\cos\theta = \dfrac{273^2 + 178^2 - 235^2}{2(273)(178)}$

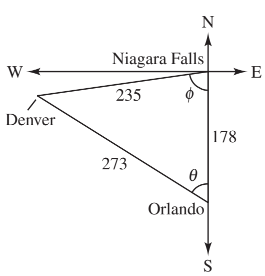
$\theta \approx 58.4^\circ$

Bearing: N 58.4° W

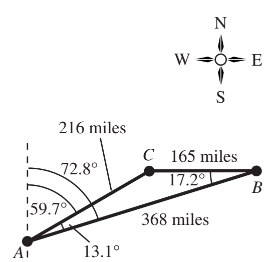
(b) $\cos\phi = \dfrac{235^2 + 178^2 - 273^2}{2(235)(178)}$

$\phi \approx 81.5^\circ$

Bearing: S 81.5° W



38.



$a = 165, b = 216, c = 368$

$$\cos B = \frac{165^2 + 368^2 - 216^2}{2(165)(368)} \approx 0.9551$$

$B \approx 17.2^\circ$

$$\cos A = \frac{216^2 + 368^2 - 165^2}{2(216)(368)} \approx 0.9741$$

$A \approx 13.1^\circ$

(a) Bearing of Minneapolis (C) from Phoenix (A)

N $(90^\circ - 17.2^\circ - 13.1^\circ)$ E

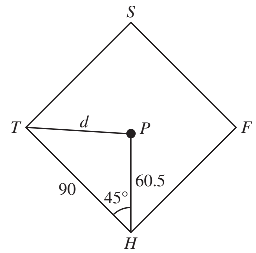
N 59.7 E

(b) Bearing of Albany (B) from Phoenix (A)

N $(90^\circ - 17.2^\circ)$ E

N 72.8° E

39. $d^2 = 60.5^2 + 90^2 - 2(60.5)(90)\cos 45° \approx 4059.8572 \implies d \approx 63.7$ ft



40. $d = \sqrt{330^2 + 420^2 - 2(330)(420)\cos 8°}$

≈ 103.9 feet

41. $a^2 = 35^2 + 20^2 - 2(35)(20)\cos 42° \implies a \approx 24.2$ miles

42. $a = \sqrt{20^2 + 20^2 - 2(20)(20)\cos 11°}$

≈ 3.8 miles

43. $\overline{RS} = \sqrt{8^2 + 10^2} = \sqrt{164} = 2\sqrt{41} \approx 12.8$ ft

$\overline{PQ} = \frac{1}{2}\sqrt{16^2 + 10^2} = \frac{1}{2}\sqrt{356} = \sqrt{89} \approx 9.4$ ft

$\tan P = \dfrac{10}{16} = \dfrac{\overline{QS}}{\overline{PS}} = \dfrac{\overline{QS}}{8} \implies \overline{QS} = 5$

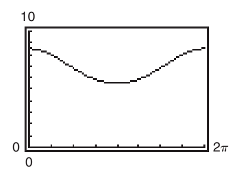
44. (a)

$$7^2 = 1.5^2 + x^2 - 2(1.5)\,x\cos\theta$$
$$49 = 2.25 + x^2 - 3x\cos\theta$$
$$x^2 - 3x\cos\theta - 46.75 = 0$$

(b) $x = \dfrac{3\cos\theta \pm \sqrt{(-3\cos\theta)^2 - 4(1)(-46.75)}}{2(1)}$

$x = \dfrac{1}{2}\left(3\cos\theta + \sqrt{9\cos^2\theta + 187}\right)$

(c)



(d) Maximum: 8.5 inches

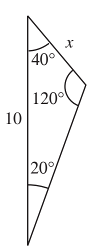
45. $d^2 = 10^2 + 7^2 - 2(10)(7)\cos\theta$

$\theta = \arccos\left[\dfrac{10^2 + 7^2 - d^2}{2(10)(7)}\right]$

$s = \dfrac{360° - \theta}{360°}(2\pi r) = \dfrac{(360° - \theta)\pi}{45°}$

d (inches)	9	10	12	13	14	15	16
θ (degrees)	60.9°	69.5°	88.0°	98.2°	109.6°	122.9°	139.8°
s (inches)	20.88	20.28	18.99	18.28	17.48	16.55	15.37

46.



$\dfrac{x}{\sin 20°} = \dfrac{10}{\sin 120°}$

$x = \dfrac{10\sin 20°}{\sin 120°} \approx 3.95$ feet

47. $a = 200$

$b = 500$

$c = 600 \implies s = \dfrac{200 + 500 + 600}{2} = 650$

Area $= \sqrt{650(450)(150)(50)} \approx 46{,}837.5$ square feet

48. area $= 2\left[\frac{1}{2}(70)(100)\sin 70°\right]$

≈ 6577.8 square meters

(The area of the parallelogram is the sum of the areas of two triangles.)

49. $s = \frac{510 + 840 + 1120}{2} = 1235$

Area $= \sqrt{1235(1235 - 510)(1235 - 840)(1235 - 1120)}$

$\approx 201{,}674$ square yards

Cost $\approx \left(\frac{201{,}674}{4840}\right)(2000) \approx \$83{,}336.36$

50. area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{(a+b+c)}{2} = \frac{(2490 + 1860 + 1350)}{2} = 2850$

area $= \sqrt{(2850(360)(990)(1500)} = 1234346.0 \text{ ft}^2$

$\frac{1234346.0 \text{ ft}^2}{(43560 \text{ ft}^2/\text{acre})} = 28.33669$ acre

$(28.33669 \text{ acre})(\$2200/\text{acre}) = \$62{,}340.71$

51. False. The average of the three sides of a triangle is

$\frac{a+b+c}{3}$, not $\frac{a+b+c}{2} = s$.

52. False. To solve an SSA triangle, the Law of Sines is needed.

53. False. If $a = 10$, $b = 16$, and $c = 5$, then by the Law of Cosines, we would have:

$$\cos A = \frac{16^2 + 5^2 - 10^2}{2(16)(5)} = 1.13125 > 1$$

This is not possible. In general, if the sum of any two sides is less than the third side, then they cannot form a triangle. Here $10 + 5$ is less than 16.

54. (a) Working with $\triangle ODC$, we have $\cos\alpha = \frac{a/2}{R}$.

This implies that $2R = \frac{a}{\cos\alpha}$.

Since we know that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

we can complete the proof by showing that $\cos\alpha = \sin A$. The solution of the system

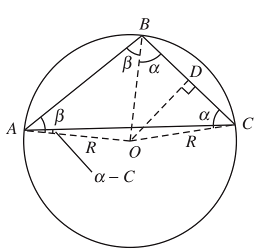
$A + B + C = 180°$

$\alpha - C + A = \beta$

$\alpha + \beta = B$

is $\alpha = 90° - A$. Therefore:

$$2R = \frac{a}{\cos\alpha} = \frac{a}{\cos(90° - A)} = \frac{a}{\sin A}.$$



(b) By Heron's Formula, the area of the triangle is

Area $= \sqrt{s(s-a)(s-b)(s-c)}$.

We can also find the area by dividing the area into six triangles and using the fact that the area is $\frac{1}{2}$ the base times the height. Using the figure as given, we have

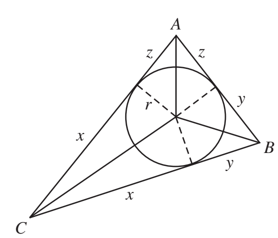
Area $= \frac{1}{2}xr + \frac{1}{2}xr + \frac{1}{2}yr + \frac{1}{2}yr + \frac{1}{2}zr + \frac{1}{2}zr$

$= r(x + y + z)$

$= rs.$

Therefore: $rs = \sqrt{s(s-a)(s-b)(s-c)} \implies$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$



55. $a = 25, b = 55, c = 72$

(a) Area of triangle: $s = \frac{1}{2}(25 + 55 + 72) = 76$

Area $= \sqrt{76(51)(21)(4)} \approx 570.60$

(b) Area of circumscribed circle:

$\cos C = \frac{25^2 + 55^2 - 72^2}{2(25)(55)} \approx -0.5578 \implies C \approx 123.9°$

$R = \frac{1}{2}\left(\frac{c}{\sin C}\right) \approx 43.37$ (see #54)

Area $= \pi R^2 \approx 5909.2$

(c) Area of inscribed circle:

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$= \sqrt{\frac{(51)(21)(4)}{76}} \approx 7.51 \quad \text{(see \#54)}$$

Area $= \pi r^2 \approx 177.09$

56. Given: $a = 200$ ft, $b = 250$ ft, $c = 325$ ft

$$s = \frac{200 + 250 + 325}{2} \approx 387.5$$

Radius of the inscribed circle: $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} = \sqrt{\frac{(187.5)(137.5)(62.5)}{387.5}} \approx 64.5$ ft (see #54)

Circumference of an inscribed circle: $C = 2\pi r \approx 2\pi(64.5) \approx 405.2$ ft

57.
$$\begin{aligned}\frac{1}{2}bc(1 + \cos A) &= \frac{1}{2}bc\left[1 + \frac{b^2 + c^2 - a^2}{2bc}\right]\\ &= \frac{1}{2}bc\left[\frac{2bc + b^2 + c^2 - a^2}{2bc}\right]\\ &= \frac{1}{4}[(b+c)^2 - a^2]\\ &= \frac{1}{4}[(b+c)+a][(b+c)-a]\\ &= \frac{b+c+a}{2}\cdot\frac{b+c-a}{2}\\ &= \frac{a+b+c}{2}\cdot\frac{-a+b+c}{2}\end{aligned}$$

58.
$$\begin{aligned}\frac{1}{2}bc(1 - \cos A) &= \frac{1}{2}bc\left[1 + \frac{a^2 - (b^2 + c^2)}{2bc}\right]\\ &= \frac{1}{2}bc\left[\frac{2bc + a^2 - b^2 - c^2}{2bc}\right]\\ &= \frac{a^2 - (b^2 - 2bc + c^2)}{4}\\ &= \frac{a^2 - (b-c)^2}{4}\\ &= \left(\frac{a - (b-c)}{2}\right)\left(\frac{a + (b-c)}{2}\right)\\ &= \frac{a-b+c}{2}\cdot\frac{a+b-c}{2}\end{aligned}$$

59. $\arcsin(-1) = -\frac{\pi}{2}$

60. $\arccos 0 = \frac{\pi}{2}$

61. $\arctan\sqrt{3} = \frac{\pi}{3}$

62. $\arctan\left(-\sqrt{3}\right) = -\arctan\sqrt{3}$
$= -\frac{\pi}{3}$

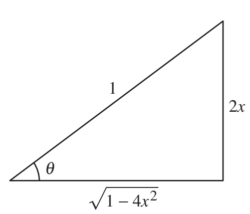
63. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

64. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \arccos\frac{\sqrt{3}}{2}$
$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

65. Let $\theta = \arcsin 2x$, then

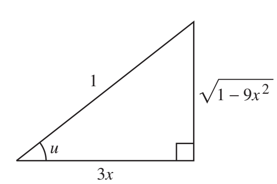
$\sin\theta = 2x = \frac{2x}{1}$ and

$\sec\theta = \frac{1}{\sqrt{1 - 4x^2}}$.



66. Let $u = \arccos 3x$

$\cos u = 3x = \frac{3x}{1}$.

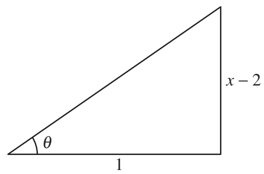


$\tan(\arccos 3x) = \tan u = \frac{\sqrt{1 - 9x^2}}{3x}$

67. Let $\theta = \arctan(x - 2)$, then

$\tan\theta = x - 2 = \dfrac{x-2}{1}$ and

$\cot\theta = \dfrac{1}{x-2}$.

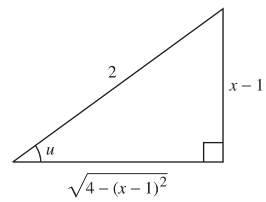


68. Let $u = \arcsin \dfrac{x-1}{2}$

$\sin u = \dfrac{x-1}{2}$.

$\cos\left(\arcsin \dfrac{x-1}{2}\right) = \cos u$

$= \dfrac{\sqrt{4-(x-1)^2}}{2}$



69. $5 = \sqrt{25 - x^2}, x = 5\sin\theta$

$5 = \sqrt{25 - (5\sin\theta)^2}$

$5 = \sqrt{25(1-\sin^2\theta)}$

$5 = 5\cos\theta$

$\cos\theta = 1$

$\sec\theta = \dfrac{1}{\cos\theta} = 1$

$\csc\theta$ is undefined.

70. $x = 2\cos\theta, -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$-\sqrt{2} = \sqrt{4 - x^2}$

$-\sqrt{2} = \sqrt{4 - (2\cos\theta)^2}$

$-\sqrt{2} = \sqrt{4 - 4\cos^2\theta}$

$-\sqrt{2} = \sqrt{4(1-\cos^2\theta)}$

$-\sqrt{2} = \sqrt{4\sin^2\theta}$

$-\sqrt{2} = 2\sin\theta$

$-\dfrac{\sqrt{2}}{2} = \sin\theta \implies \cos\theta = \dfrac{\sqrt{2}}{2}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\sqrt{2}/2} = \sqrt{2}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\sqrt{2}/2} = -\sqrt{2}$

71. $-\sqrt{3} = \sqrt{x^2 - 9}, x = 3\sec\theta$

$-\sqrt{3} = \sqrt{(3\sec\theta)^2 - 9}$

$-\sqrt{3} = \sqrt{9(\sec^2\theta - 1)}$

$-\sqrt{3} = 3\tan\theta$

$\tan\theta = -\dfrac{\sqrt{3}}{3}$

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1 + \left(-\dfrac{\sqrt{3}}{3}\right)^2} = \dfrac{2\sqrt{3}}{3}$

$\cot\theta = \dfrac{1}{\tan\theta} = -\sqrt{3}$

$\csc\theta = -\sqrt{1+\cot^2\theta} = -\sqrt{1 + (-\sqrt{3})^2} = -2$

72. $x = 6\tan\theta, -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$12 = \sqrt{36 + x^2}$

$12 = \sqrt{36 + (6\tan\theta)^2}$

$12 = \sqrt{36 + 36\tan^2\theta}$

$12 = \sqrt{36(1+\tan^2\theta)}$

$12 = \sqrt{36\sec^2\theta}$

$12 = 6\sec\theta$

$2 = \sec\theta$

$\cos\theta = \dfrac{1}{2}$

$\sin^2\theta + \left(\dfrac{1}{2}\right)^2 = 1$

$\sin^2\theta = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\sin\theta = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\pm\sqrt{3}/2} = \pm\dfrac{2}{\sqrt{3}} = \pm\dfrac{2\sqrt{3}}{3}$

73. $\cos\frac{5\pi}{6} - \cos\frac{\pi}{3} = -2\sin\left(\frac{\frac{5\pi}{6}+\frac{\pi}{3}}{2}\right)\sin\left(\frac{\frac{5\pi}{6}-\frac{\pi}{3}}{2}\right) = -2\sin\frac{7\pi}{12}\sin\frac{\pi}{4}$

74. $\sin\left(x-\frac{\pi}{2}\right) - \sin\left(x+\frac{\pi}{2}\right) = 2\cos\left(\frac{x-\frac{\pi}{2}+x+\frac{\pi}{2}}{2}\right)\sin\left(\frac{x-\frac{\pi}{2}-\left(x+\frac{\pi}{2}\right)}{2}\right)$

$$= 2\cos\left(\frac{2x}{2}\right)\sin\left(\frac{-\pi}{2}\right)$$

$$= 2\cos x\sin\left(-\frac{\pi}{2}\right)$$

Section 6.3 Vectors in the Plane

- A vector $\mathbf{v}$ is the collection of all directed line segments that are equivalent to a given directed line segment $\overrightarrow{PQ}$.
- You should be able to *geometrically* perform the operations of vector addition and scalar multiplication.
- The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

  $\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2\rangle = \langle v_1, v_2\rangle = \mathbf{v}$.
- The magnitude of $\mathbf{v} = \langle v_1, v_2\rangle$ is given by $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.
- If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ is a unit vector.
- You should be able to perform the operations of scalar multiplication and vector addition in component form.

  (a) $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2\rangle$ (b) $k\mathbf{u} = \langle ku_1, ku_2\rangle$
- You should know the following properties of vector addition and scalar multiplication.

  (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
  (b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
  (c) $\mathbf{u} + \mathbf{0} = \mathbf{u}$
  (d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
  (e) $c(d\mathbf{u}) = (cd)\mathbf{u}$
  (f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
  (g) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
  (h) $1(\mathbf{u}) = \mathbf{u}$, $0\mathbf{u} = \mathbf{0}$
  (i) $\|c\mathbf{v}\| = |c|\,\|\mathbf{v}\|$
- A unit vector in the direction of $\mathbf{v}$ is $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$.
- The standard unit vectors are $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$. $\mathbf{v} = \langle v_1, v_2\rangle$ can be written as $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$.
- A vector $\mathbf{v}$ with magnitude $\|\mathbf{v}\|$ and direction θ can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|(\cos\theta)\mathbf{i} + \|\mathbf{v}\|(\sin\theta)\mathbf{j}$, where $\tan\theta = b/a$.

Vocabulary Check

1. directed line segment
2. initial; terminal
3. magnitude
4. vector
5. standard position
6. unit vector
7. multiplication; addition
8. resultant
9. linear combination; horizontal; vertical

1. $\mathbf{v} = \langle 4 - 0, 1 - 0 \rangle = \langle 4, 1 \rangle$

$\mathbf{u} = \mathbf{v}$

2. $\mathbf{u} = \langle -3 - 0, -4 - 4 \rangle = \langle -3, -8 \rangle$

$\mathbf{v} = \langle 0 - 3, -5 - 3 \rangle = \langle -3, -8 \rangle$

$\mathbf{u} = \mathbf{v}$

3. Initial point: $(0, 0)$

Terminal point: $(3, 2)$

$\mathbf{v} = \langle 3 - 0, 2 - 0 \rangle = \langle 3, 2 \rangle$

$\|\mathbf{v}\| = \sqrt{3^2 + 2^2} = \sqrt{13}$

4. Initial point: $(0, 0)$

Terminal point: $(-4, -2)$

$\mathbf{v} = \langle -4 - 0, -2 - 0 \rangle = \langle -4, -2 \rangle$

$\|\mathbf{v}\| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$

5. Initial point: $(2, 2)$

Terminal point: $(-1, 4)$

$\mathbf{v} = \langle -1 - 2, 4 - 2 \rangle = \langle -3, 2 \rangle$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$

6. Initial point: $(-1, -1)$

Terminal point: $(3, 5)$

$\mathbf{v} = \langle 3 - (-1), 5 - (-1) \rangle = \langle 4, 6 \rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$

7. Initial point: $(3, -2)$

Terminal point: $(3, 3)$

$\mathbf{v} = \langle 3 - 3, 3 - (-2) \rangle = \langle 0, 5 \rangle$

$\|\mathbf{v}\| = \sqrt{0^2 + 5^2} = \sqrt{25} = 5$

8. Initial point: $(-4, -1)$

Terminal point: $(3, -1)$

$\mathbf{v} = \langle 3 - (-4), -1 - (-1) \rangle = \langle 7, 0 \rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + 0^2} = 7$

9. Initial point: $(-1, 5)$

Terminal point: $(15, 12)$

$\mathbf{v} = \langle 15 - (-1), 12 - 5 \rangle = \langle 16, 7 \rangle$

$\|\mathbf{v}\| = \sqrt{16^2 + 7^2} = \sqrt{305}$

10. Initial point: $(1, 11)$

Terminal point: $(9, 3)$

$\mathbf{v} = \langle 9 - 1, 3 - 11 \rangle = \langle 8, -8 \rangle$

$\|\mathbf{v}\| = \sqrt{8^2 + (-8)^2} = 8\sqrt{2}$

11. Initial point: $(-3, -5)$

Terminal point: $(5, 1)$

$\mathbf{v} = \langle 5 - (-3), 1 - (-5) \rangle = \langle 8, 6 \rangle$

$\|\mathbf{v}\| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

12. Initial point: $(-3, 11)$

Terminal point: $(9, 40)$

$\mathbf{v} = \langle 9 - (-3), 40 - 11 \rangle = \langle 12, 29 \rangle$

$\|\mathbf{v}\| = \sqrt{12^2 + 29^2} = \sqrt{985}$

13. Initial point: $(1, 3)$

Terminal point: $(-8, -9)$

$\mathbf{v} = \langle -8 - 1, -9 - 3 \rangle = \langle -9, -12 \rangle$

$\|\mathbf{v}\| = \sqrt{(-9)^2 + (-12)^2} = \sqrt{225} = 15$

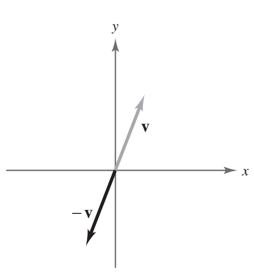
14. Initial point: $(-2, 7)$

Terminal point: $(5, -17)$

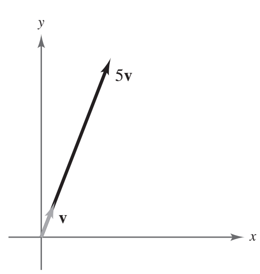
$\mathbf{v} = \langle 5 - (-2), -17 - 7 \rangle = \langle 7, -24 \rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + (-24)^2} = 25$

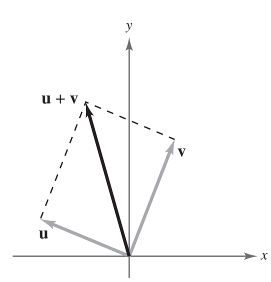
15.



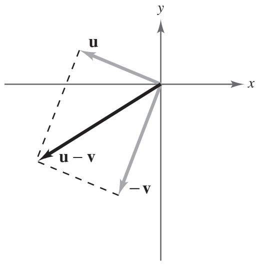
16. $5\mathbf{v}$



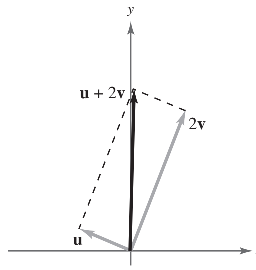
17.



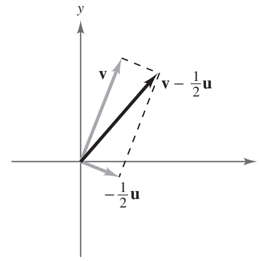
18. $\mathbf{u} - \mathbf{v}$



19. $\mathbf{u} + 2\mathbf{v}$

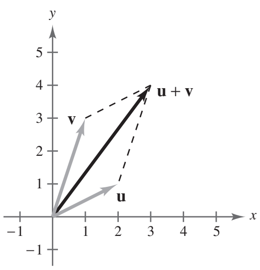


20. $\mathbf{v} - \frac{1}{2}\mathbf{u}$

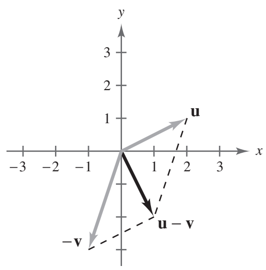


21. $\mathbf{u} = \langle 2, 1 \rangle$, $\mathbf{v} = \langle 1, 3 \rangle$

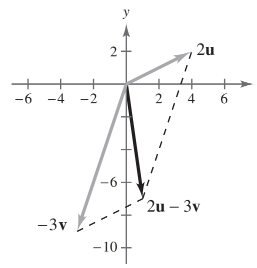
(a) $\mathbf{u} + \mathbf{v} = \langle 3, 4 \rangle$



(b) $\mathbf{u} - \mathbf{v} = \langle 1, -2 \rangle$

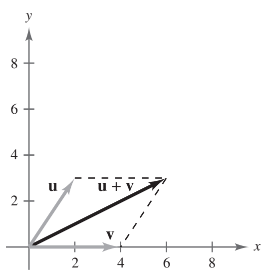


(c) $2\mathbf{u} - 3\mathbf{v} = \langle 4, 2 \rangle - \langle 3, 9 \rangle = \langle 1, -7 \rangle$

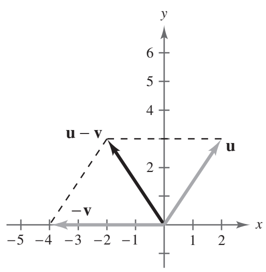


22. $\mathbf{u} = \langle 2, 3 \rangle$, $\mathbf{v} = \langle 4, 0 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 6, 3 \rangle$

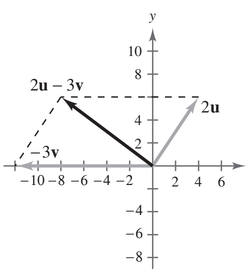


(b) $\mathbf{u} - \mathbf{v} = \langle -2, 3 \rangle$



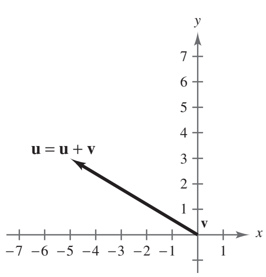
(c) $2\mathbf{u} - 3\mathbf{v} = \langle 4, 6 \rangle - \langle 12, 0 \rangle$

$= \langle -8, 6 \rangle$

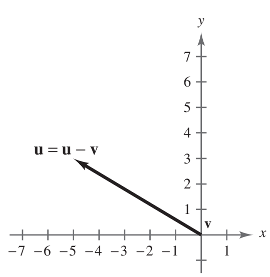


23. $\mathbf{u} = \langle -5, 3 \rangle$, $\mathbf{v} = \langle 0, 0 \rangle$

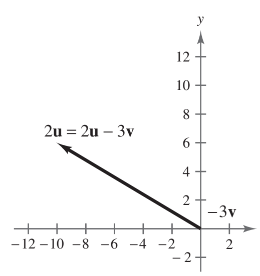
(a) $\mathbf{u} + \mathbf{v} = \langle -5, 3 \rangle = \mathbf{u}$



(b) $\mathbf{u} - \mathbf{v} = \langle -5, 3 \rangle = \mathbf{u}$

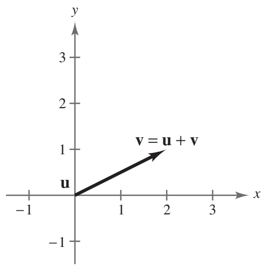


(c) $2\mathbf{u} - 3\mathbf{v} = 2\mathbf{u} = \langle -10, 6 \rangle$

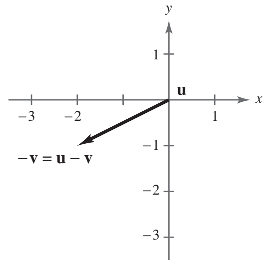


24. $\mathbf{u} = \langle 0, 0\rangle, \mathbf{v} = \langle 2, 1\rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 2, 1\rangle$

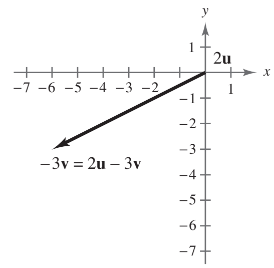


(b) $\mathbf{u} - \mathbf{v} = \langle -2, -1\rangle$



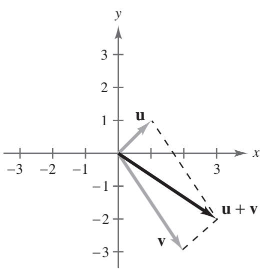
(c) $2\mathbf{u} - 3\mathbf{v} = \langle 0, 0\rangle - \langle 6, 3\rangle$

$= \langle -6, -3\rangle$

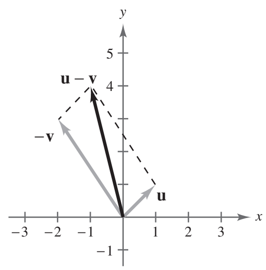


25. $\mathbf{u} = \mathbf{i} + \mathbf{j}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

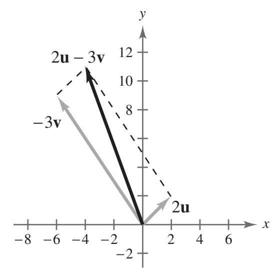


(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} + 4\mathbf{j}$



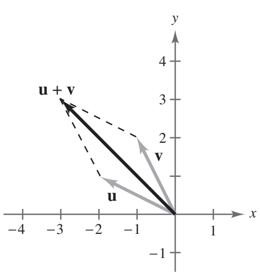
(c) $2\mathbf{u} - 3\mathbf{v} = (2\mathbf{i} + 2\mathbf{j}) - (6\mathbf{i} - 9\mathbf{j})$

$= -4\mathbf{i} + 11\mathbf{j}$

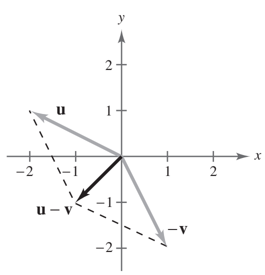


26. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}, \mathbf{v} = -\mathbf{i} + 2\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = -3\mathbf{i} + 3\mathbf{j}$

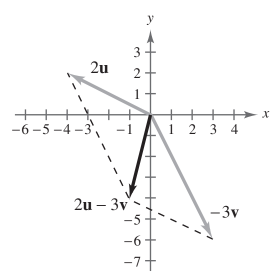


(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} - \mathbf{j}$



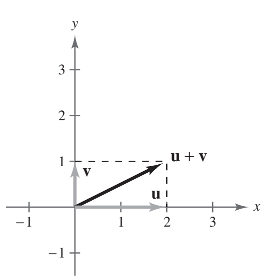
(c) $2\mathbf{u} - 3\mathbf{v} = (-4\mathbf{i} + 2\mathbf{j}) - (-3\mathbf{i} + 6\mathbf{j})$

$= -\mathbf{i} - 4\mathbf{j}$

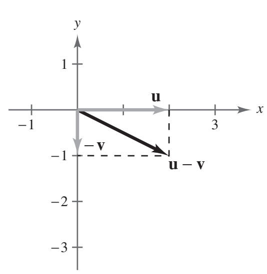


27. $\mathbf{u} = 2\mathbf{i}, \mathbf{v} = \mathbf{j}$

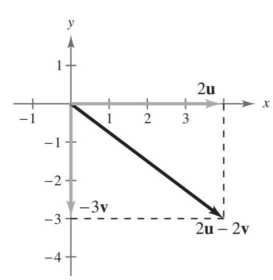
(a) $\mathbf{u} + \mathbf{v} = 2\mathbf{i} + \mathbf{j}$



(b) $\mathbf{u} - \mathbf{v} = 2\mathbf{i} - \mathbf{j}$

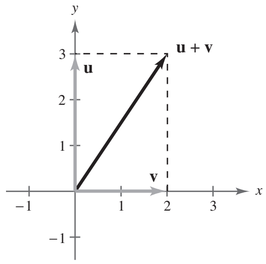


(c) $2\mathbf{u} - 3\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$

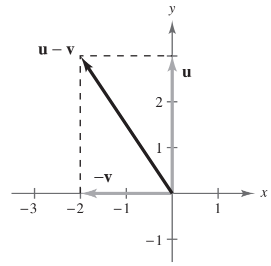


28. $\mathbf{u} = 3\mathbf{j}, \mathbf{v} = 2\mathbf{i}$

(a) $\mathbf{u} + \mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$

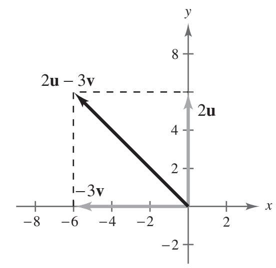


(b) $\mathbf{u} - \mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$



(c) $2\mathbf{u} - 3\mathbf{v} = 6\mathbf{j} - 6\mathbf{i}$

$= -6\mathbf{i} + 6\mathbf{j}$



29. $\mathbf{v} = \dfrac{1}{\|\mathbf{u}\|}\mathbf{u} = \dfrac{1}{\sqrt{3^2 + 0^2}}\langle 3, 0\rangle = \dfrac{1}{3}\langle 3, 0\rangle = \langle 1, 0\rangle$

30. $\mathbf{u} = \langle 0, -2\rangle$

$$\mathbf{v} = \frac{1}{\|\mathbf{u}\|}\mathbf{u} = \frac{1}{\sqrt{0^2 + (-2)^2}}\langle 0, -2\rangle$$
$$= \frac{1}{2}\langle 0, -2\rangle = \langle 0, -1\rangle$$

31. $\mathbf{u} = \dfrac{1}{\|\mathbf{v}\|}\mathbf{v} = \dfrac{1}{\sqrt{(-2)^2 + 2^2}}\langle -2, 2\rangle = \dfrac{1}{2\sqrt{2}}\langle -2, 2\rangle$

$$= \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$$
$$= \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$$

32. $\mathbf{v} = \langle 5, -12\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{5^2 + (-12)^2}}\langle 5, -12\rangle$$
$$= \frac{1}{13}\langle 5, -12\rangle$$
$$= \left\langle \frac{5}{13}, -\frac{12}{13}\right\rangle$$

33. $\mathbf{u} = \dfrac{1}{\|\mathbf{v}\|}\mathbf{v} = \dfrac{1}{\sqrt{6^2 + (-2)^2}}(6\mathbf{i} - 2\mathbf{j}) = \dfrac{1}{\sqrt{40}}(6\mathbf{i} - 2\mathbf{j})$

$$= \frac{1}{2\sqrt{10}}(6\mathbf{i} - 2\mathbf{j}) = \frac{3}{\sqrt{10}}\mathbf{i} - \frac{1}{\sqrt{10}}\mathbf{j}$$
$$= \frac{3\sqrt{10}}{10}\mathbf{i} - \frac{\sqrt{10}}{10}\mathbf{j}$$

34. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$
$$= \frac{1}{\sqrt{1^2 + 1^2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

35. $\mathbf{u} = \dfrac{1}{\|\mathbf{w}\|}\mathbf{w} = \dfrac{1}{4}(4\mathbf{j}) = \mathbf{j}$

36. $\mathbf{w} = -6\mathbf{i}$

$$\mathbf{v} = \frac{1}{\|\mathbf{w}\|}\mathbf{w} = \frac{1}{\sqrt{(-6)^2 + 0^2}}(-6\mathbf{i})$$
$$= \frac{1}{6}(-6\mathbf{i}) = -\mathbf{i}$$

37. $\mathbf{u} = \dfrac{1}{\|\mathbf{w}\|}\mathbf{w} = \dfrac{1}{\sqrt{1^2 + (-2)^2}}(\mathbf{i} - 2\mathbf{j}) = \dfrac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$

$$= \frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j} = \frac{\sqrt{5}}{5}\mathbf{i} - \frac{2\sqrt{5}}{5}\mathbf{j}$$

38. $\mathbf{w} = 7\mathbf{j} - 3\mathbf{i}$

$$\mathbf{v} = \frac{1}{\|\mathbf{w}\|}\mathbf{w} = \frac{1}{\sqrt{(-3)^2 + 7^2}}(-3\mathbf{i} + 7\mathbf{j})$$
$$= -\frac{3}{\sqrt{58}}\mathbf{i} + \frac{7}{\sqrt{58}}\mathbf{j} = -\frac{3\sqrt{58}}{58}\mathbf{i} + \frac{7\sqrt{58}}{58}\mathbf{j}$$

39. $5\left(\dfrac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 5\left(\dfrac{1}{\sqrt{3^2+3^2}}\langle 3, 3\rangle\right) = \dfrac{5}{3\sqrt{2}}\langle 3, 3\rangle$

$= \left\langle \dfrac{5}{\sqrt{2}}, \dfrac{5}{\sqrt{2}} \right\rangle = \left\langle \dfrac{5\sqrt{2}}{2}, \dfrac{5\sqrt{2}}{2} \right\rangle$

40. $\mathbf{v} = 6\left(\dfrac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 6\left(\dfrac{1}{\sqrt{(-3)^2+3^2}}\langle -3, 3\rangle\right)$

$= 6\left(\dfrac{1}{3\sqrt{2}}\langle -3, 3\rangle\right) = \left\langle -\dfrac{6}{\sqrt{2}}, \dfrac{6}{\sqrt{2}} \right\rangle = \left\langle -3\sqrt{2}, 3\sqrt{2} \right\rangle$

41. $9\left(\dfrac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 9\left(\dfrac{1}{\sqrt{2^2+5^2}}\langle 2, 5\rangle\right) = \dfrac{9}{\sqrt{29}}\langle 2, 5\rangle$

$= \left\langle \dfrac{18}{\sqrt{29}}, \dfrac{45}{\sqrt{29}} \right\rangle = \left\langle \dfrac{18\sqrt{29}}{29}, \dfrac{45\sqrt{29}}{29} \right\rangle$

42. $\mathbf{v} = 10\left(\dfrac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 10\left(\dfrac{1}{\sqrt{0^2+(-10)^2}}\langle -10, 0\rangle\right)$

$= 10\left(\dfrac{1}{10}\langle -10, 0\rangle\right) = \langle -10, 0\rangle$

43. $\mathbf{u} = \langle 4 - (-3), 5 - 1\rangle$

$= \langle 7, 4\rangle$

$= 7\mathbf{i} + 4\mathbf{j}$

44. $\mathbf{u} = \langle 3 - 0, 6 - (-2)\rangle$

$\mathbf{u} = \langle 3, 8\rangle$

$\mathbf{u} = 3\mathbf{i} + 8\mathbf{j}$

45. $\mathbf{u} = \langle 2 - (-1), 3 - (-5)\rangle$

$= \langle 3, 8\rangle$

$= 3\mathbf{i} + 8\mathbf{j}$

46. $\mathbf{u} = \langle 0 - (-6), 1 - 4\rangle$

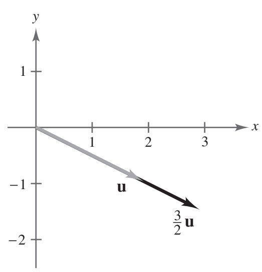
$\mathbf{u} = \langle 6, -3\rangle$

$\mathbf{u} = 6\mathbf{i} - 3\mathbf{j}$

47. $\mathbf{v} = \frac{3}{2}\mathbf{u}$

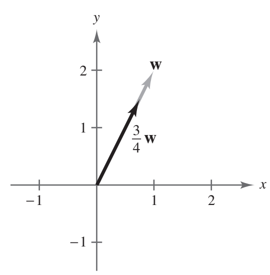
$= \frac{3}{2}(2\mathbf{i} - \mathbf{j})$

$= 3\mathbf{i} - \frac{3}{2}\mathbf{j} = \left\langle 3, -\frac{3}{2}\right\rangle$



48. $\mathbf{v} = \frac{3}{4}\mathbf{w} = \frac{3}{4}(\mathbf{i} + 2\mathbf{j})$

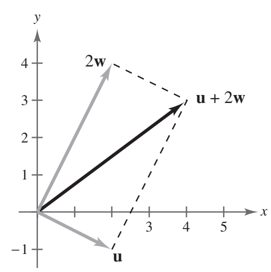
$= \frac{3}{4}\mathbf{i} + \frac{3}{2}\mathbf{j} = \left\langle \frac{3}{4}, \frac{3}{2}\right\rangle$



49. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$

$= (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j})$

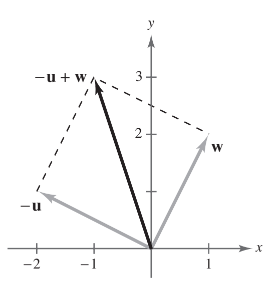
$= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3\rangle$



50. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

$= -(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$

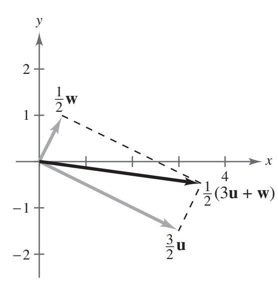
$= -\mathbf{i} + 3\mathbf{j} = \langle -1, 3\rangle$



51. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

$= \frac{1}{2}(6\mathbf{i} - 3\mathbf{j} + \mathbf{i} + 2\mathbf{j})$

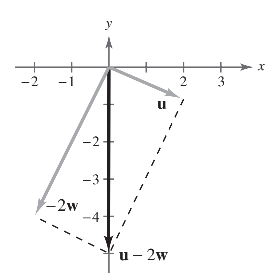
$= \frac{7}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} = \left\langle \frac{7}{2}, -\frac{1}{2}\right\rangle$



52. $\mathbf{v} = \mathbf{u} - 2\mathbf{w}$

$= (2\mathbf{i} - \mathbf{j}) - 2(\mathbf{i} + 2\mathbf{j})$

$= -5\mathbf{j} = \langle 0, -5\rangle$



53. $\mathbf{v} = 3(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$\|\mathbf{v}\| = 3, \ \theta = 60°$

54. $\mathbf{v} = 8(\cos 135°\,\mathbf{i} + \sin 135°\,\mathbf{j})$

$\|\mathbf{v}\| = 8, \ \theta = 135°$

55. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$$

$$\tan \theta = \frac{-6}{6} = -1$$

Since $\mathbf{v}$ lies in Quadrant IV, $\theta = 315°$.

56. $\mathbf{v} = -5\mathbf{i} + 4\mathbf{j}$

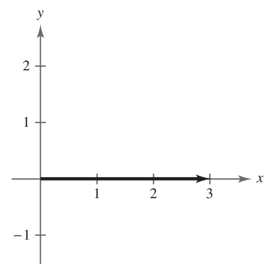
$$\|\mathbf{v}\| = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$$

$$\tan \theta = -\frac{4}{5}$$

Since $\mathbf{v}$ lies in Quadrant II, $\theta = 141.3°$.

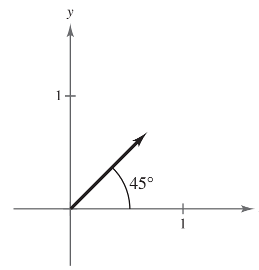
57. $\mathbf{v} = \langle 3\cos 0°, 3\sin 0°\rangle$

$= \langle 3, 0\rangle$



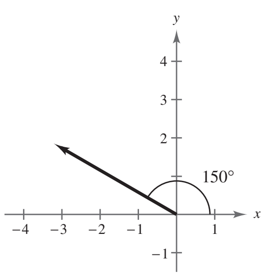
58. $\mathbf{v} = \langle \cos 45°, \sin 45°\rangle$

$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$$



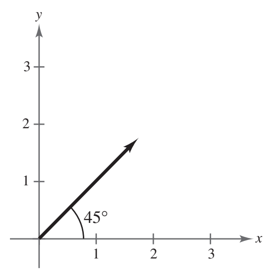
59. $\mathbf{v} = \left\langle \frac{7}{2}\cos 150°, \frac{7}{2}\sin 150°\right\rangle$

$$= \left\langle -\frac{7\sqrt{3}}{4}, \frac{7}{4}\right\rangle$$



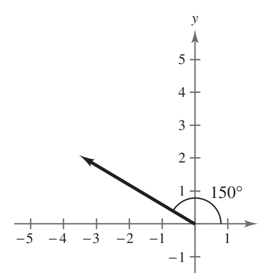
60. $\mathbf{v} = \left\langle \frac{5}{2}\cos 45°, \frac{5}{2}\sin 45°\right\rangle$

$$= \left\langle \frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{4}\right\rangle$$



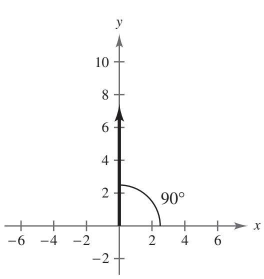
61. $\mathbf{v} = \langle 3\sqrt{2}\cos 150°, \ 3\sqrt{2}\sin 150°\rangle$

$$= \left\langle -\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2}\right\rangle$$



62. $\mathbf{v} = \left\langle 4\sqrt{3}\cos 90°, 4\sqrt{3}\sin 90°\right\rangle$

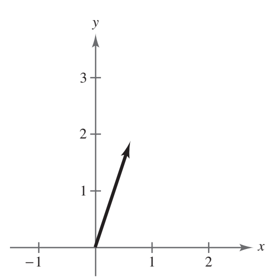
$= \left\langle 0, 4\sqrt{3}\right\rangle$



63. $\mathbf{v} = 2\left(\frac{1}{\sqrt{1^2 + 3^2}}\right)(\mathbf{i} + 3\mathbf{j})$

$$= \frac{2}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j})$$

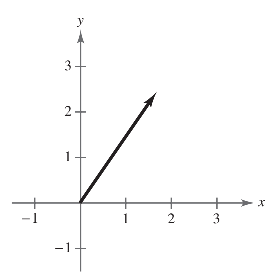
$$= \frac{\sqrt{10}}{5}\mathbf{i} + \frac{3\sqrt{10}}{5}\mathbf{j} = \left\langle \frac{\sqrt{10}}{5}, \frac{3\sqrt{10}}{5}\right\rangle$$



64. $\mathbf{v} = 3\left(\frac{1}{\sqrt{3^2 + 4^2}}\right)(3\mathbf{i} + 4\mathbf{j})$

$$= \frac{3}{5}(3\mathbf{i} + 4\mathbf{j})$$

$$= \frac{9}{5}\mathbf{i} + \frac{12}{5}\mathbf{j} = \left\langle \frac{9}{5}, \frac{12}{5}\right\rangle$$



65. $\mathbf{u} = \langle 5\cos 0°, 5\sin 0°\rangle = \langle 5, 0\rangle$

$\mathbf{v} = \langle 5\cos 90°, 5\sin 90°\rangle = \langle 0, 5\rangle$

$\mathbf{u} + \mathbf{v} = \langle 5, 5\rangle$

66. $\mathbf{u} = \langle 4\cos 60°, 4\sin 60°\rangle = \left\langle 2, 2\sqrt{3}\right\rangle$

$\mathbf{v} = \langle 4\cos 90°, 4\sin 90°\rangle = \langle 0, 4\rangle$

$\mathbf{u} + \mathbf{v} = \left\langle 2, 4 + 2\sqrt{3}\right\rangle$

67. $\mathbf{u} = \langle 20\cos 45°, 20\sin 45°\rangle = \left\langle 10\sqrt{2}, 10\sqrt{2}\right\rangle$

$\mathbf{v} = \langle 50\cos 180°, 50\sin 180°\rangle = \langle -50, 0\rangle$

$\mathbf{u} + \mathbf{v} = \left\langle 10\sqrt{2} - 50, 10\sqrt{2}\right\rangle$

68. $\mathbf{u} = \langle 50\cos 30°, 50\sin 30°\rangle = \left\langle 25\sqrt{3}, 25\right\rangle$

$\approx \langle 43.301, 25\rangle$

$\mathbf{v} = \langle 30\cos 110°, 30\sin 110°\rangle \approx \langle -10.261, 28.191\rangle$

$\mathbf{u} + \mathbf{v} \approx \langle 33.04, 53.19\rangle$

69. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

$\mathbf{w} = 2\mathbf{i} - 2\mathbf{j}$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$

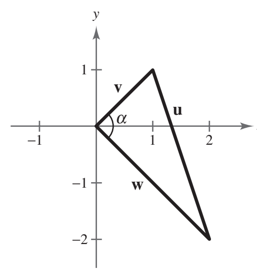
$\|\mathbf{v}\| = \sqrt{2}$

$\|\mathbf{w}\| = 2\sqrt{2}$

$\|\mathbf{v} - \mathbf{w}\| = \sqrt{10}$

$$\cos\alpha = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \frac{2 + 8 - 10}{2\sqrt{2} \cdot 2\sqrt{2}} = 0$$

$\alpha = 90°$



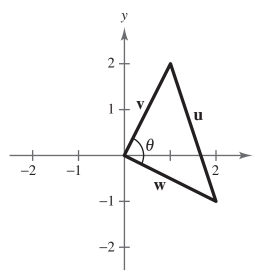
70. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$\mathbf{w} = 2\mathbf{i} - \mathbf{j}$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \frac{5 + 5 - 10}{2\sqrt{5}\sqrt{5}} = 0$$

$\theta = 90°$



71. Force One: $\mathbf{u} = 45\mathbf{i}$

Force Two: $\mathbf{v} = 60\cos\theta\mathbf{i} + 60\sin\theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (45 + 60\cos\theta)\mathbf{i} + 60\sin\theta\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(45 + 60\cos\theta)^2 + (60\sin\theta)^2} = 90$

$$2025 + 5400\cos\theta + 3600 = 8100$$

$$5400\cos\theta = 2475$$

$$\cos\theta = \frac{2475}{5400} \approx 0.4583$$

$$\theta \approx 62.7°$$

72. Force One: $\mathbf{u} = 3000\mathbf{i}$

Force Two: $\mathbf{v} = 1000\cos\theta\mathbf{i} + 1000\sin\theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (3000 + 1000\cos\theta)\mathbf{i} + 1000\sin\theta\mathbf{j}$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(3000 + 1000\cos\theta)^2 + (1000\sin\theta)^2} = 3750$$

$$9{,}000{,}000 + 6{,}000{,}000\cos\theta + 1{,}000{,}000 = 14{,}062,500$$

$$6{,}000{,}000\cos\theta = 4{,}062,500$$

$$\cos\theta = \frac{4{,}062{,}500}{6{,}000{,}000} \approx 0.6771$$

$$\theta \approx 47.4°$$

73. $\mathbf{u} = 300\mathbf{i}$

$$\mathbf{v} = (125\cos 45°)\mathbf{i} + (125\sin 45°)\mathbf{j} = \frac{125}{\sqrt{2}}\mathbf{i} + \frac{125}{\sqrt{2}}\mathbf{j}$$

$$\mathrm{R} = \mathbf{u} + \mathbf{v} = \left(300 + \frac{125}{\sqrt{2}}\right)\mathbf{i} + \frac{125}{\sqrt{2}}\mathbf{j}$$

$$\|\mathrm{R}\| = \sqrt{\left(300 + \frac{125}{\sqrt{2}}\right)^2 + \left(\frac{125}{\sqrt{2}}\right)^2} \approx 398.32 \text{ newtons}$$

$$\tan\theta = \frac{\frac{125}{\sqrt{2}}}{300 + \left(\frac{125}{\sqrt{2}}\right)} \implies \theta \approx 12.8°$$

74. $\mathbf{u} = (2000\cos 30°)\,\mathbf{i} + (2000\sin 30°\mathbf{j})$

$$\approx 1732.05\mathbf{i} + 1000\mathbf{j}$$

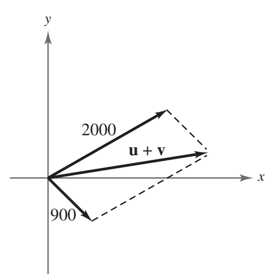
$$\mathbf{v} = (900\cos(-45°))\mathbf{i} + (900\sin(-45°)\mathbf{j})$$

$$\approx 636.4\mathbf{i} + -636.4\mathbf{j}$$

$$\mathbf{u} + \mathbf{v} \approx 2368.4\,\mathbf{i} + 363.6\mathbf{j}$$

$$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(2368.4)^2 + (363.6)^2} \approx 2396.19$$

$$\tan\theta = \frac{363.6}{2368.4} \approx 0.1535 \implies \theta \approx 8.7°$$



75. $\mathbf{u} = (75\cos 30°)\mathbf{i} + (75\sin 30°)\mathbf{j} \approx 64.95\mathbf{i} + 37.5\mathbf{j}$

$$\mathbf{v} = (100\cos 45°)\mathbf{i} + (100\sin 45°)\mathbf{j} \approx 70.71\mathbf{i} + 70.71\mathbf{j}$$

$$\mathbf{w} = (125\cos 120°)\mathbf{i} + (125\sin 120°)\mathbf{j} \approx -62.5\mathbf{i} + 108.3\mathbf{j}$$

$$\mathbf{u} + \mathbf{v} + \mathbf{w} \approx 73.16\mathbf{i} + 216.5\mathbf{j}$$

$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \approx 228.5 \text{ pounds}$$

$$\tan\theta \approx \frac{216.5}{73.16} \approx 2.9593$$

$$\theta \approx 71.3°$$

76.

$$\mathbf{u} = (70\cos 30°)\mathbf{i} - (70\sin 30°)\mathbf{j} \approx 60.62\mathbf{i} - 35\mathbf{j}$$

$$\mathbf{v} = (40\cos 45°)\mathbf{i} + (40\sin 45°)\mathbf{j} \approx 28.28\mathbf{i} + 28.28\mathbf{j}$$

$$\mathbf{w} = (60\cos 135°)\mathbf{i} + (60\sin 135°)\mathbf{j} \approx -42.43\mathbf{i} + 42.43\mathbf{j}$$

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = 46.48\mathbf{i} + 35.71\mathbf{j}$$

$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \approx 58.61 \text{ pounds}$$

$$\tan\theta \approx \frac{35.71}{46.47} \approx 0.7683$$

$$\theta \approx 37.5°$$

77. Horizontal component of velocity: $70\cos 35° \approx 57.34$ feet per second

Vertical component of velocity: $70\sin 35° \approx 40.15$ feet per second

78. Horizontal component of velocity: $1200\cos 6° \approx 1193.4$ ft/sec

Vertical component of velocity: $1200\sin 6° \approx 125.4$ ft/sec

79. Cable $\overrightarrow{AC}$: $\mathbf{u} = \|\mathbf{u}\|(\cos 50°\mathbf{i} - \sin 50°\mathbf{j})$

Cable $\overrightarrow{BC}$: $\mathbf{v} = \|\mathbf{v}\|(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = -2000\mathbf{j}$

$$\|\mathbf{u}\|\cos 50° - \|\mathbf{v}\|\cos 30° = 0$$

$$-\|\mathbf{u}\|\sin 50° - \|\mathbf{v}\|\sin 30° = -2000$$

Solving this system of equations yields:

$$T_{AC} = \|\mathbf{u}\| \approx 1758.8 \text{ pounds}$$

$$T_{BC} = \|\mathbf{v}\| \approx 1305.4 \text{ pounds}$$

80. Rope $\overrightarrow{AC}$: $\mathbf{u} = 10\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant IV and its reference angle is $\arctan\left(\frac{12}{5}\right)$.

$$\mathbf{u} = \|\mathbf{u}\|\left[\cos\left(\arctan \tfrac{12}{5}\right)\mathbf{i} - \sin\left(\arctan \tfrac{12}{5}\right)\mathbf{j}\right]$$

Rope $\overrightarrow{BC}$: $\mathbf{v} = -20\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant III and its reference angle is $\arctan\left(\frac{6}{5}\right)$.

$$\mathbf{v} = \|\mathbf{v}\|\left[-\cos\left(\arctan \tfrac{6}{5}\right)\mathbf{i} - \sin\left(\arctan \tfrac{6}{5}\right)\mathbf{j}\right]$$

Resultant: $\mathbf{u} + \mathbf{v} = -5000\mathbf{j}$

$$\|\mathbf{u}\|\cos\left(\arctan \tfrac{12}{5}\right) - \|\mathbf{v}\|\cos\left(\arctan \tfrac{6}{5}\right) = 0$$

$$-\|\mathbf{u}\|\sin\left(\arctan \tfrac{12}{5}\right) - \|\mathbf{v}\|\sin\left(\arctan \tfrac{6}{5}\right) = -5000$$

Solving this system of equations yields: $T_{AC} = \|\mathbf{u}\| \approx 3611.1$ pounds

$T_{BC} = \|\mathbf{v}\| \approx 2169.5$ pounds

81. Towline 1: $\mathbf{u} = \|\mathbf{u}\|(\cos 18°\mathbf{i} + \sin 18°\mathbf{j})$

Towline 2: $\mathbf{v} = \|\mathbf{u}\|(\cos 18°\mathbf{i} - \sin 18°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = 6000\mathbf{i}$

$$\|\mathbf{u}\|\cos 18° + \|\mathbf{u}\|\cos 18° = 6000$$

$$\|\mathbf{u}\| \approx 3154.4$$

Therefore, the tension on each towline is $\|\mathbf{u}\| \approx 3154.4$ pounds.

82. Rope 1: $\mathbf{u} = \|\mathbf{u}\|(\cos 70°\mathbf{i} - \sin 70°\mathbf{j})$

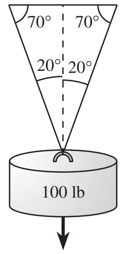
Rope 2: $\mathbf{v} = \|\mathbf{u}\|(-\cos 70°\mathbf{i} - \sin 70°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = -100\mathbf{j}$

$$-\|\mathbf{u}\| \sin 70° - \|\mathbf{u}\| \sin 70° = -100$$

$$\|\mathbf{u}\| \approx 53.2$$

Therefore, the tension of each rope is $\|\mathbf{u}\| \approx 53.2$ pounds.



83. Airspeed: $\mathbf{u} = (875 \cos 58°)\mathbf{i} - (875 \sin 58°)\mathbf{j}$

Groundspeed: $\mathbf{v} = (800 \cos 50°)\mathbf{i} - (800 \sin 50°)\mathbf{j}$

Wind: $\mathbf{w} = \mathbf{v} - \mathbf{u} = (800 \cos 50° - 875 \cos 58°)\mathbf{i} + (-800 \sin 50° + 875 \sin 58°)\mathbf{j}$

$$\approx 50.5507\mathbf{i} + 129.2065\mathbf{j}$$

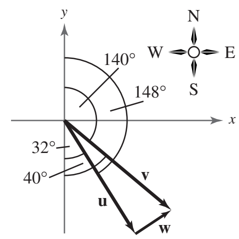
Wind speed: $\|\mathbf{w}\| \approx \sqrt{(50.5507)^2 + (129.2065)^2}$

≈ 138.7 kilometers per hour

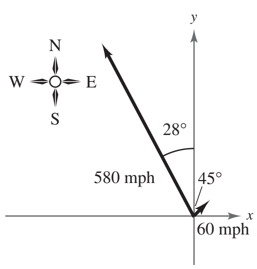
Wind direction: $\tan \theta \approx \dfrac{129.2065}{50.5507}$

$$\theta \approx 68.6°;\ 90° - \theta = 21.4°$$

Bearing: N 21.4° E



84. (a)



(b) The velocity vector $\mathbf{v}_w$ of the wind has a magnitude of 60 and a direction angle of 45°.

$$\mathbf{v}_w = \|\mathbf{v}_w\|(\cos \theta)\mathbf{i} + \|\mathbf{v}_w\|(\sin \theta)\mathbf{j}$$
$$= 60(\cos 45°)\mathbf{i} + 60(\sin 45°)\mathbf{j}$$
$$= 60[(\cos 45°)\mathbf{i} + (\sin 45°)\mathbf{j}]$$
$$= 60\langle \cos 45°, \sin 45° \rangle, \text{ or } \langle 30\sqrt{2}, 30\sqrt{2} \rangle$$

(c) The velocity vector $\mathbf{v}_j$ of the jet has a magnitude of 580 and a direction angle of 118°.

$$\mathbf{v}_j = \|\mathbf{v}_j\|(\cos \theta)\mathbf{i} + \|\mathbf{v}_j\|(\sin \theta)\mathbf{j}$$
$$= 580(\cos 118°)\mathbf{i} + 580(\sin 118°)\mathbf{j}$$
$$= 580[(\cos 118°)\mathbf{i} + (\sin 118°)\mathbf{j}]$$
$$= 580\langle \cos 118°, \sin 118° \rangle$$

—CONTINUED—

84. —CONTINUED—

(d) The velocity of the jet (in the wind) is

$$\begin{aligned}\mathbf{v} &= \mathbf{v}_w + \mathbf{v}_j\\ &= 60\langle \cos 45°, \sin 45°\rangle + 580\langle \cos 118°, \sin 118°\rangle\\ &= \langle 60\cos 45° + 580\cos 118°, 60\sin 45° + 580\sin 118°\rangle\\ &\approx \langle -229.87, 554.54\rangle\end{aligned}$$

The resultant speed of the jet is

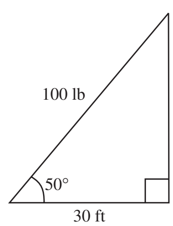
$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(-229.87)^2 + (554.54)^2}\\ &\approx 600.3 \text{ miles per hour}\end{aligned}$$

(e) If θ is the direction of the flight path, then

$$\tan\theta = \frac{554.54}{-229.87} \approx -2.4124$$

Because θ lies in the Quadrant II, $\theta = 180° + \arctan(-2.4124) \approx 180° - 67.5° = 112.5°$. The true bearing of the jet is $112.5° - 90° = 22.5°$ west of north, or $360° - 22.5° = 337.5°$.

85. $W = FD = (100\ \cos 50°)(30) = 1928.4$ foot–pounds



86. Horizontal force: $\mathbf{u} = \|\mathbf{u}\|\mathbf{i}$

Weight: $\mathbf{w} = -\mathbf{j}$

Rope: $\mathbf{t} = \|\mathbf{t}\|(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

$$\mathbf{u} + \mathbf{w} + \mathbf{t} = \mathbf{0} \implies \|\mathbf{u}\| + \|\mathbf{t}\|\cos 135° = 0$$
$$-1 + \|\mathbf{t}\|\sin 135° = 0$$

$\|\mathbf{t}\| \approx \sqrt{2}$ pounds

$\|\mathbf{u}\| \approx 1$ pound

87. True. See Example 1.

88. True.

$\|\mathbf{u}\| = \sqrt{a^2 + b^2} = 1 \implies a^2 + b^2 = 1$

89. (a) The angle between them is 0°.

(b) The angle between them is 180°.

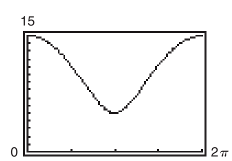
(c) No. At most it can be equal to the sum when the angle between them is 0°.

90. $\mathbf{F}_1 = \langle 10, 0\rangle, \mathbf{F}_2 = 5\langle \cos\theta, \sin\theta\rangle$

(a) $\mathbf{F}_1 + \mathbf{F}_2 = \langle 10 + 5\cos\theta, 5\sin\theta\rangle$

$$\begin{aligned}\|\mathbf{F}_1 + \mathbf{F}_2\| &= \sqrt{(10 + 5\cos\theta)^2 + (5\sin\theta)^2}\\ &= \sqrt{100 + 100\cos\theta + 25\cos^2\theta + 25\sin^2\theta}\\ &= 5\sqrt{4 + 4\cos\theta + \cos^2\theta + \sin^2\theta}\\ &= 5\sqrt{4 + 4\cos\theta + 1}\\ &= 5\sqrt{5 + 4\cos\theta}\end{aligned}$$

(b)



(c) Range: $[5, 15]$
Maximum is 15 when $\theta = 0$.
Minimum is 5 when $\theta = \pi$.

(d) The magnitude of the resultant is never 0 because the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are not the same.

91. Let $\mathbf{v} = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$.

$\|\mathbf{v}\| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$

Therefore, $\mathbf{v}$ is a unit vector for any value of θ.

92. The following program is written for a *TI-82* or *TI-83* or *TI-83 Plus* graphing calculator. The program sketches two vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$ in standard position, and then sketches the vector difference $\mathbf{u} - \mathbf{v}$ using the parallelogram law.

```
PROGRAM: SUBVECT
:Input "ENTER A", A
:Input "ENTER B", B
:Input "ENTER C", C
:Input "ENTER D", D
:Line (0, 0, A, B)
:Line (0, 0, C, D)
:Pause
:A – C→E
:B – D→F
:Line (A, B, C, D)
:Line (A, B, E, F)
:Line (0, 0, E, F)
:Pause
:ClrDraw
:Stop
```

93. $\mathbf{u} = \langle 5 - 1, 2 - 6\rangle = \langle 4, -4\rangle$

$\mathbf{v} = \langle 9 - 4, 4 - 5\rangle = \langle 5, -1\rangle$

$\mathbf{u} - \mathbf{v} = \langle -1, -3\rangle$ or $\mathbf{v} - \mathbf{u} = \langle 1, 3\rangle$

94. $\mathbf{u} = \langle 80 - 10, 80 - 60\rangle = \langle 70, 20\rangle$

$\mathbf{v} = \langle -20 - (-100), 70 - 0\rangle = \langle 80, 70\rangle$

$\mathbf{u} - \mathbf{v} = \langle 70 - 80, 20 - 70\rangle = \langle -10, -50\rangle$

$\mathbf{v} - \mathbf{u} = \langle 80 - 70, 70 - 20\rangle = \langle 10, 50\rangle$

95.
$$\begin{aligned}\sqrt{x^2 - 64} &= \sqrt{(8\sec\theta)^2 - 64}\\ &= \sqrt{64(\sec^2\theta - 1)}\\ &= 8\sqrt{\tan^2\theta}\\ &= 8\tan\theta \quad \text{for} \quad 0 < \theta < \frac{\pi}{2}\end{aligned}$$

96. $x = 8\sin\theta$
$$\begin{aligned}\sqrt{64 - x^2} &= \sqrt{64 - (8\sin^2\theta)}\\ &= \sqrt{64 - 64\sin^2\theta}\\ &= 8\sqrt{1 - \sin^2\theta}\\ &= 8\sqrt{\cos^2\theta}\\ &= 8\cos\theta \text{ for } 0 < \theta < \frac{\pi}{2}\end{aligned}$$

97.
$$\begin{aligned}\sqrt{x^2 + 36} &= \sqrt{(6\tan\theta)^2 + 36}\\ &= \sqrt{36(\tan^2\theta + 1)}\\ &= 6\sqrt{\sec^2\theta}\\ &= 6\sec\theta \quad \text{for} \quad 0 < \theta < \frac{\pi}{2}\end{aligned}$$

98. $x = 5\sec\theta$
$$\begin{aligned}\sqrt{(x^2 - 25)^3} &= \sqrt{[(5\sec\theta)^2 - 25]^3}\\ &= \sqrt{(25\sec^2\theta - 25)^3}\\ &= \sqrt{[25(\sec^2\theta - 1)]^3}\\ &= \sqrt{(25\tan^2\theta)^3}\\ &= \sqrt{15{,}625\tan^6\theta}\\ &= 125\tan^3\theta \text{ for } 0 < \theta < \frac{\pi}{2}\end{aligned}$$

99. $\cos x(\cos x + 1) = 0$

$\cos x = 0 \quad \text{or} \quad \cos x + 1 = 0$

$x = \dfrac{\pi}{2} + n\pi \qquad \cos x = -1$

$x = \pi + 2n\pi$

100. $\sin x(2\sin x + \sqrt{2}) = 0$

$\sin x = 0 \qquad 2\sin x + \sqrt{2} = 0$

$x = 0 + n\pi \qquad \sin x = -\dfrac{\sqrt{2}}{2}$

$x = \dfrac{5\pi}{4} + 2\pi n, \dfrac{7\pi}{4} + 2\pi n$

$x = n\pi, \dfrac{5\pi}{4} + 2\pi n, \dfrac{7\pi}{4} + 2\pi n$

101. $3\sec x\sin x - 2\sqrt{3}\sin x = 0$

$\sin x(3\sec x - 2\sqrt{3}) = 0$

$\sin x = 0$ or $3\sec x - 2\sqrt{3} = 0$

$x = n\pi \qquad \sec x = \dfrac{2\sqrt{3}}{3}$

$\cos x = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{6} + 2n\pi$

$x = \dfrac{11\pi}{6} + 2n\pi$

102. $\cos x\csc x + \cos x\sqrt{2} = 0$

$\cos x(\csc x + \sqrt{2}) = 0$

$\cos x = 0 \qquad \csc x + \sqrt{2} = 0$

$x = \dfrac{\pi}{2} + n\pi \qquad \csc x = -\sqrt{2}$

$x = \dfrac{5\pi}{4} + 2\pi n, \dfrac{7\pi}{4} + 2\pi n$

$x = \dfrac{\pi}{2} + n\pi, \dfrac{5\pi}{4} + 2\pi n, \dfrac{7\pi}{4} + 2\pi n$

Section 6.4 Vectors and Dot Products

- Know the definition of the dot product of $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$.

  $\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2$
- Know the following properties of the dot product:
  1. $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$
  2. $\mathbf{0}\cdot\mathbf{v} = 0$
  3. $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) = \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}$
  4. $\mathbf{v}\cdot\mathbf{v} = \|\mathbf{v}\|^2$
  5. $c(\mathbf{u}\cdot\mathbf{v}) = c\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot c\mathbf{v}$
- If θ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

  $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$
- The vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $\mathbf{u}\cdot\mathbf{v} = 0$.
- Know the definition of vector components.

  $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$ where $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal, and $\mathbf{w}_1$ is parallel to $\mathbf{v}$. $\mathbf{w}_1$ is called the projection of $\mathbf{u}$ onto $\mathbf{v}$ and is denoted by $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$. Then we have $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1$.
- Know the definition of work.
  1. Projection form: $w = \|\text{proj}_{\overrightarrow{PQ}}\,\mathbf{F}\|\,\|\overrightarrow{PQ}\|$
  2. Dot product form: $w = \mathbf{F}\cdot\overrightarrow{PQ}$

Vocabulary Check

1. dot product
2. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$
3. orthogonal
4. $\left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$
5. $\|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \, \|\overrightarrow{PQ}\|$; $\mathbf{F} \cdot \overrightarrow{PQ}$

1. $\mathbf{u} = \langle 6, 1 \rangle, \ \mathbf{v} = \langle -2, 3 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 6(-2) + 1(3) = -9$

2. $\mathbf{u} = \langle 5, 12 \rangle, \mathbf{v} = \langle -3, 2 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 5(-3) + 12(2) = 9$

3. $\mathbf{u} = \langle -4, 1 \rangle, \mathbf{v} = \langle 2, -3 \rangle$

$\mathbf{u} \cdot \mathbf{v} = -4(2) + 1(-3) = -11$

4. $\mathbf{u} = \langle -2, 5 \rangle, \mathbf{v} = \langle -1, -2 \rangle$

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= (-2)(-1) + 5(-2) \\ &= 2 - 10 = -8\end{aligned}$$

5. $\mathbf{u} = 4\mathbf{i} - 2\mathbf{j}, \ \mathbf{v} = \mathbf{i} - \mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 4(1) + (-2)(-1) = 6$

6. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = 7\mathbf{i} - 2\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 3(7) + 4(-2) = 13$

7. $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}, \mathbf{v} = -2\mathbf{i} - 3\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 3(-2) + 2(-3) = -12$

8. $\mathbf{u} = \mathbf{i} - 2\mathbf{j}, \mathbf{v} = -2\mathbf{i} + \mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 1(-2) + (-2)1 = -4$

9. $\mathbf{u} = \langle 2, 2 \rangle$

$\mathbf{u} \cdot \mathbf{u} = 2(2) + 2(2) = 8$

The result is a scalar.

10. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle$

$3\mathbf{u} \cdot \mathbf{v} = 3[(2)(-3) + (2)(4)] = 3(2) = 6$

The result is a scalar.

11. $\mathbf{u} = \langle 2, 2 \rangle, \ \mathbf{v} = \langle -3, 4 \rangle$

$$\begin{aligned}(\mathbf{u} \cdot \mathbf{v})\mathbf{v} &= [(2)(-3) + 2(4)]\langle -3, 4 \rangle \\ &= 2\langle -3, 4 \rangle = \langle -6, 8 \rangle\end{aligned}$$

The result is a vector.

12. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle, \mathbf{w} = \langle 1, -2 \rangle$

$$\begin{aligned}(\mathbf{v} \cdot \mathbf{u})\mathbf{w} &= [(-3)(2) + (4)(2)]\langle 1, -2 \rangle \\ &= 2\langle 1, -2 \rangle \\ &= \langle 2, -4 \rangle \text{ vector}\end{aligned}$$

13. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle, \mathbf{w} = \langle 1, -2 \rangle$

$$\begin{aligned}(3\mathbf{w} \cdot \mathbf{v})\mathbf{u} &= [3(1)(-3) + 3(-2)(4)]\langle 2, 2 \rangle \\ &= -33\langle 2, 2 \rangle \\ &= \langle -66, -66 \rangle\end{aligned}$$

The result is a vector.

14. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle, \mathbf{w} = \langle 1, -2 \rangle$

$2\mathbf{v} = \langle -6, 8 \rangle$

$$\begin{aligned}(\mathbf{u} \cdot 2\mathbf{v})\mathbf{w} &= [(2)(-6) + (2)(8)]\langle 1, -2 \rangle \\ &= 4\langle 1, -2 \rangle \\ &= \langle 4, -8 \rangle \text{ vector}\end{aligned}$$

15. $\mathbf{w} = \langle 1, -2 \rangle$

$\|\mathbf{w}\| - 1 = \sqrt{(1)^2 + (-2)^2} - 1 = \sqrt{5} - 1$

The result is a scalar.

16. $\mathbf{u} = \langle 2, 2 \rangle$

$$\begin{aligned}2 - \|\mathbf{u}\| &= 2 - \sqrt{\mathbf{u} \cdot \mathbf{u}} \\ &= 2 - \sqrt{(2)(2) + (2)(2)} \\ &= 2 - \sqrt{8} \\ &= 2 - 2\sqrt{2} \ \text{ scalar}\end{aligned}$$

17. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle, \mathbf{w} = \langle 1, -2 \rangle$

$$\begin{aligned}(\mathbf{u} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{w}) &= [2(-3) + 2(4)] - [2(1) + 2(-2)] \\ &= 2 - (-2) \\ &= 4\end{aligned}$$

The result is a scalar.

18. $\mathbf{u} = \langle 2, 2\rangle, \mathbf{v} = \langle -3, 4\rangle, \mathbf{w} = \langle 1, -2\rangle$

$$\begin{aligned}(\mathbf{v}\cdot\mathbf{u}) - (\mathbf{w}\cdot\mathbf{v}) &= [(-3)(2) + (4)(2)] - [(1)(-3) + (-2)(4)]\\ &= 2 - (-11)\\ &= 13 \text{ scalar}\end{aligned}$$

19. $\mathbf{u} = \langle -5, 12\rangle$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(-5)^2 + 12^2} = 13$$

20. $\mathbf{u} = \langle 2, -4\rangle$

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{\mathbf{u}\cdot\mathbf{u}}\\ &= \sqrt{2(2) + (-4)(-4)}\\ &= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

21. $\mathbf{u} = 20\mathbf{i} + 25\mathbf{j}$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(20)^2 + (25)^2} = \sqrt{1025} = 5\sqrt{41}$$

22. $\mathbf{u} = 12\mathbf{i} - 16\mathbf{j}$

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{12(12) + (-16)(-16)}\\ &= \sqrt{400} = 20\end{aligned}$$

23. $\mathbf{u} = 6\mathbf{j}$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6$$

24. $\mathbf{u} = -21\mathbf{i}$

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(-21)(-21) + 0(0)}\\ &= \sqrt{21^2} = 21\end{aligned}$$

25. $\mathbf{u} = \langle 1, 0\rangle,\ \mathbf{v} = \langle 0, -2\rangle$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{0}{(1)(2)} = 0$$

$$\theta = 90^\circ$$

26. $\mathbf{u} = \langle 3, 2\rangle, \mathbf{v} = \langle 4, 0\rangle$

$$\begin{aligned}\cos\theta &= \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{3(4) + 2(0)}{\sqrt{13}\,(4)}\\ &= \frac{3}{\sqrt{13}} \approx 0.83205\end{aligned}$$

$$\theta \approx 33.69^\circ$$

27. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j},\ \mathbf{v} = -2\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = -\frac{8}{(5)(2)}$$

$$\theta = \arccos\left(-\frac{4}{5}\right)$$

$$\theta \approx 143.13^\circ$$

28. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$$\begin{aligned}\cos\theta &= \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}\\ &= \frac{2(1) + (-3)(-2)}{\sqrt{2^2 + 3^2}\sqrt{1^2 + 2^2}}\\ &= \frac{8}{\sqrt{65}} \approx 0.992278\end{aligned}$$

$$\theta \approx 7.13^\circ$$

29. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = 6\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{8}{\sqrt{5}\sqrt{52}} = 0.4961$$

$$\theta = 60.26^\circ$$

30. $\mathbf{u} = -6\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$

$$\cos\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-6(-8) + (-3)(4)}{\sqrt{45}\sqrt{80}} = \frac{36}{60} = 0.6$$

$$\theta \approx 53.13^\circ$$

31. $\mathbf{u} = 5\mathbf{i} + 5\mathbf{j},\ \mathbf{v} = -6\mathbf{i} + 6\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = 0$$

$$\theta = 90^\circ$$

32. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{2(4) + (-3)(3)}{\sqrt{13}\sqrt{25}} \approx -0.0555$$

$$\theta \approx 93.18^\circ$$

33. $\mathbf{u} = \left(\cos\frac{\pi}{3}\right)\mathbf{i} + \left(\sin\frac{\pi}{3}\right)\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$

$\mathbf{v} = \left(\cos\frac{3\pi}{4}\right)\mathbf{i} + \left(\sin\frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\|\mathbf{u}\| = \|\mathbf{v}\| = 1$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \mathbf{u}\cdot\mathbf{v}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2} + \sqrt{6}}{4}$$

$$\theta = \arccos\left(\frac{-\sqrt{2} + \sqrt{6}}{4}\right) = 75° = \frac{5\pi}{12}$$

34. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{v} = \cos\left(\frac{\pi}{2}\right)\mathbf{i} + \sin\left(\frac{\pi}{2}\right)\mathbf{j} = \mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\frac{\sqrt{2}}{2}(0) + \frac{\sqrt{2}}{2}(1)}{1\cdot 1} = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}$$

35. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$

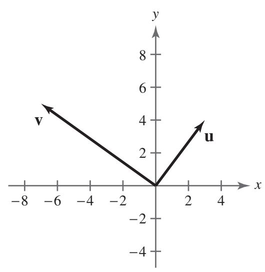
$\mathbf{v} = -7\mathbf{i} + 5\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{3(-7) + 4(5)}{3\sqrt{74}}$$

$$= \frac{-1}{5\sqrt{74}} \approx -0.0232$$

$\theta \approx 91.3°$



36. $\mathbf{u} = 6\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} + 4\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta$

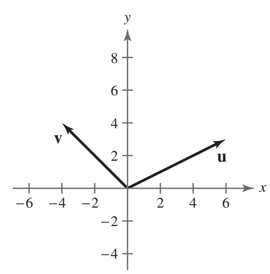
$$(6)(-4) + (3)(4) = \sqrt{6^2 + 3^2}\cdot\sqrt{(-4)^2 + (4)^2}\cdot\cos\theta$$

$$-12 = \sqrt{45}\cdot\sqrt{32}\cos\theta$$

$$-12 = 12\sqrt{10}\cos\theta$$

$$\frac{-1}{\sqrt{10}} = \cos\theta$$

$$\cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) = \theta \implies \theta \approx 108.4°$$



37. $\mathbf{u} = 5\mathbf{i} + 5\mathbf{j}$

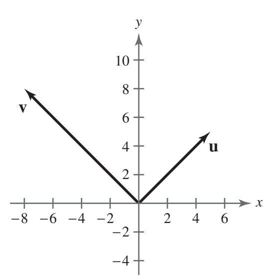
$\mathbf{v} = -8\mathbf{i} + 8\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{5(-8) + 5(8)}{\sqrt{50}\sqrt{128}}$$

$$= 0$$

$\theta = 90°$



38. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = 8\mathbf{i} + 3\mathbf{j}$

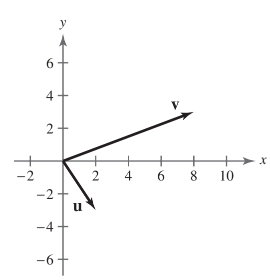
$\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta$

$$(2)(8) + (-3)(3) = \sqrt{2^2 + (-3)^2}\cdot\sqrt{8^2 + 3^2}\cdot\cos\theta$$

$$7 = \sqrt{13}\cdot\sqrt{73}\cos\theta$$

$$\frac{7}{\sqrt{13}\cdot\sqrt{73}} = \cos\theta$$

$$\cos^{-1}\left[\frac{7}{\sqrt{13}\cdot\sqrt{73}}\right] = \theta \implies \theta \approx 76.9°$$



39. $P = (1, 2),\ Q = (3, 4),\ R = (2, 5)$

$\overrightarrow{PQ} = \langle 2, 2\rangle,\ \overrightarrow{PR} = \langle 1, 3\rangle,\ \overrightarrow{QR} = \langle -1, 1\rangle$

$\cos\alpha = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{PR}\|} = \dfrac{8}{(2\sqrt{2})(\sqrt{10})} \implies \alpha = \arccos\dfrac{2}{\sqrt{5}} \approx 26.57°$

$\cos\beta = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{QR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{QR}\|} = 0 \implies \beta = 90°$. Thus, $\gamma = 180° - 26.57° - 90° = 63.43°$.

40. $P = (-3, -4), Q = (1, 7), R = (8, 2)$

$\overrightarrow{PQ} = \langle 4, 11\rangle, \overrightarrow{QR} = \langle 7, -5\rangle,$

$\overrightarrow{PR} = \langle 11, 6\rangle, \overrightarrow{QP} = \langle -4, -11\rangle$

$\cos\alpha = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{PR}\|} = \dfrac{110}{(\sqrt{137})(\sqrt{157})} \implies \alpha \approx 41.41°$

$\cos\beta = \dfrac{\overrightarrow{QR} \cdot \overrightarrow{QP}}{\|\overrightarrow{QR}\|\,\|\overrightarrow{QP}\|} = \dfrac{27}{(\sqrt{74})(\sqrt{137})} \implies \beta \approx 74.45°$

$\gamma \approx 180° - 41.41° - 74.45° = 64.14°$

41. $P = (-3, 0), Q = (2, 2), R = (0, 6)$

$\overrightarrow{QP} = \langle -5, -2\rangle, \overrightarrow{PR} = \langle 3, 6\rangle, \overrightarrow{QR} = \langle -2, 4\rangle, \overrightarrow{PQ} = \langle 5, 2\rangle$

$\cos\alpha = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{PR}\|} = \dfrac{27}{\sqrt{29}\sqrt{45}} \implies \alpha \approx 41.63°$

$\cos\beta = \dfrac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{\|\overrightarrow{QP}\|\,\|\overrightarrow{PR}\|} = \dfrac{2}{\sqrt{29}\sqrt{20}} \implies \beta \approx 85.24°$

$\delta = 180° - 41.63° - 85.24° = 53.13°$

42. $P = (-3, 5), Q = (-1, 9), R = (7, 9)$

$\overrightarrow{PQ} = \langle 2, 4\rangle, \overrightarrow{QR} = \langle 8, 0\rangle,$

$\overrightarrow{PR} = \langle 10, 4\rangle, \overrightarrow{QP} = \langle -2, -4\rangle$

$\cos\alpha = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{PR}\|} = \dfrac{36}{(\sqrt{20})(\sqrt{116})} \implies \alpha \approx 41.6°$

$\cos\beta = \dfrac{\overrightarrow{QR} \cdot \overrightarrow{QP}}{\|\overrightarrow{QR}\|\,\|\overrightarrow{QP}\|} = \dfrac{-16}{8(\sqrt{20})} \implies \beta \approx 116.6°$

$\gamma \approx 180° - 41.6° - 116.6° = 21.8°$

43.
$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta \\ &= (4)(10)\cos\frac{2\pi}{3} \\ &= 40\left(-\frac{1}{2}\right) \\ &= -20\end{aligned}$$

44. $\|\mathbf{u}\| = 100, \|\mathbf{v}\| = 250, \theta = \dfrac{\pi}{6}$

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta \\ &= (100)(250)\cos\frac{\pi}{6} \\ &= 25{,}000 \cdot \frac{\sqrt{3}}{2} \\ &= 12{,}500\sqrt{3}\end{aligned}$$

45.
$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta \\ &= (9)(36)\cos\frac{3\pi}{4} \\ &= 324\left(-\frac{\sqrt{2}}{2}\right) \\ &= -162\sqrt{2} \approx -229.1\end{aligned}$$

46. $\|\mathbf{u}\| = 4$

$\|\mathbf{v}\| = 12$

$\theta = \dfrac{\pi}{3}$

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta \\ &= (4)(12)\cos\frac{\pi}{3} \\ &= (4)(12)\left(\frac{1}{2}\right) = 24\end{aligned}$$

47. $\mathbf{u} = \langle -12, 30\rangle,\ \mathbf{v} = \left\langle \dfrac{1}{2}, -\dfrac{5}{4}\right\rangle$

$\mathbf{u} = -24\mathbf{v} \implies \mathbf{u}$ and $\mathbf{v}$ are parallel.

48. $\mathbf{u} = \langle 3, 15\rangle, \mathbf{v} = \langle -1, 5\rangle$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u} \cdot \mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

49. $\mathbf{u} = \dfrac{1}{4}(3\mathbf{i} - \mathbf{j}),\ \mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u} \cdot \mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

50. $\mathbf{u} = 1, \mathbf{v} = -2\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u} \cdot \mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

51. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j},\ \mathbf{v} = -\mathbf{i} - \mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ **u** and **v** are orthogonal.

52. $\mathbf{u} = \langle \cos\theta, \sin\theta \rangle$

$\mathbf{v} = \langle \sin\theta, -\cos\theta \rangle$

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ **u** and **v** are orthogonal.

53. $\mathbf{u} = \langle 2, 2 \rangle,\ \mathbf{v} = \langle 6, 1 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{14}{37}\langle 6, 1 \rangle = \frac{1}{37}\langle 84, 14 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 2 \rangle - \frac{14}{37}\langle 6, 1 \rangle = \left\langle -\frac{10}{37}, \frac{60}{37} \right\rangle = \frac{10}{37}\langle -1, 6 \rangle = \frac{1}{37}\langle -10, 60 \rangle$$

$$\mathbf{u} = \frac{1}{37}\langle 84, 14 \rangle + \frac{1}{37}\langle -10, 60 \rangle = \langle 2, 2 \rangle$$

54. $\mathbf{u} = \langle 4, 2 \rangle, \mathbf{v} = \langle 1, -2 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\langle 1, -2 \rangle = \langle 0, 0 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2 \rangle - \langle 0, 0 \rangle = \langle 4, 2 \rangle$$

$$\mathbf{u} = \langle 4, 2 \rangle + \langle 0, 0 \rangle = \langle 4, 2 \rangle$$

55. $\mathbf{u} = \langle 0, 3 \rangle,\ \mathbf{v} = \langle 2, 15 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{45}{229}\ \langle 2, 15 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3 \rangle - \frac{45}{229}\langle 2, 15 \rangle = \left\langle -\frac{90}{229}, \frac{12}{229} \right\rangle$$

$$= \frac{6}{229}\langle -15, 2 \rangle$$

$$\mathbf{u} = \frac{45}{229}\langle 2, 15 \rangle + \frac{6}{229}\langle -15, 2 \rangle = \langle 0, 3 \rangle$$

56. $\mathbf{u} = \langle -3, -2 \rangle, \mathbf{v} = \langle -4, -1 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{14}{17}\right)\langle -4, -1 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -3, -2 \rangle - \frac{14}{17}\langle -4, -1 \rangle = \frac{5}{17}\langle 1, -4 \rangle$$

$$\mathbf{u} = \frac{14}{17}\langle -4, -1 \rangle + \frac{5}{17}\langle 1, -4 \rangle = \langle -3, -2 \rangle$$

57. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{0}$ since they are perpendicular.

Since **u** and **v** are orthogonal, $\mathbf{u} \cdot \mathbf{v} = \mathbf{0}$ and $\text{proj}_{\mathbf{v}}\, \mathbf{u} = \mathbf{0}$.

$\text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = 0$, since $\mathbf{u} \cdot \mathbf{v} = \mathbf{0}$.

58. Because **u** and **v** are orthogonal, the projection of **u** onto **v** is **0**.

$\text{proj}_{\mathbf{w}}\, \mathbf{u} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \mathbf{0}$ since $\mathbf{u} \cdot \mathbf{v} = \mathbf{0}$.

59. $\mathbf{u} = \langle 3, 5 \rangle$

For **v** to be orthogonal to **u**, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\langle -5, 3 \rangle$ and $\langle 5, -3 \rangle$

60. $\mathbf{u} = \langle -8, 3 \rangle$

For **v** to be orthogonal to **u**, $\mathbf{u} \cdot \mathbf{v}$ must be equal to 0.

Two possibilities: $\langle 3, 8 \rangle, \langle -3, -8 \rangle$

61. $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j}$

For **u** and **v** to be orthogonal, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\mathbf{v} = \frac{2}{3}\mathbf{i} + \frac{1}{2}\mathbf{j}$ and $\mathbf{v} = -\frac{2}{3}\mathbf{i} - \frac{1}{2}\mathbf{j}$

62. $\mathbf{u} = -\frac{5}{2}\mathbf{i} - 3\mathbf{j}$

For **v** to be orthogonal to **u**, $\mathbf{u} \cdot \mathbf{v}$ must be equal to 0.

Two possibilities: $\mathbf{v} = 3\mathbf{i} - \frac{5}{2}\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + \frac{5}{2}\mathbf{j}$

63. $\mathbf{w} = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\|\overrightarrow{PQ}\|$ where $\overrightarrow{PQ} = \langle 4, 7\rangle$ and $\mathbf{v} = \langle 1, 4\rangle$.

$$\text{proj}_{\overrightarrow{PQ}}\mathbf{v} = \left(\frac{\mathbf{v}\cdot\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2}\right)\overrightarrow{PQ} = \left(\frac{32}{65}\right)\langle 4, 7\rangle$$

$$\mathbf{w} = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\|\overrightarrow{PQ}\| = \left(\frac{32\sqrt{65}}{65}\right)\left(\sqrt{65}\right) = 32$$

64. $P = (1, 3)$, $Q = (-3, 5)$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$

$$\begin{aligned}\text{work} &= \mathbf{v}\cdot\overrightarrow{PQ}\\ &= (-2\mathbf{i} + 3\mathbf{j})\cdot(-4\mathbf{i} + 2\mathbf{j})\\ &= (-2)(-4) + 3(2) = 14\end{aligned}$$

65. (a) $\mathbf{u} = \langle 1650, 3200\rangle$, $\mathbf{v} = \langle 15.25, 10.50\rangle$

$\mathbf{u}\cdot\mathbf{v} = 1650(15.25) + 3200(10.50) = \$58{,}762.50$

This gives the total revenue that can be earned by selling all of the pans.

(b) Increase prices by 5%: $1.05\mathbf{v}$ The operation is scalar multiplication.

$$\begin{aligned}\mathbf{u}\cdot 1.05\mathbf{v} &= 1.05\mathbf{u}\cdot\mathbf{v}\\ &= 1.05[1650(15.25) + 3200(10.50)]\\ &= 1.05(58{,}762.50)\\ &= 61{,}700.63\end{aligned}$$

66. (a) $\mathbf{u} = \langle 3240, 2450\rangle$, $\mathbf{v} = \langle 1.75, 1.25\rangle$

$\mathbf{u}\cdot\mathbf{v} = (3240)(1.75) + (2450)(1.25) = 8732.5$

The fast food stand sold \$8732.50 of hamburgers and hot dogs in one month.

(b) Increase prices by 2.5%:

$1.025\mathbf{v}$ scalar multiplication

67. (a) Force due to gravity:

$\mathbf{F} = -30{,}000\mathbf{j}$

Unit vector along hill:

$\mathbf{v} = (\cos d)\mathbf{i} + (\sin d)\mathbf{j}$

Projection of $\mathbf{F}$ onto $\mathbf{v}$:

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{F} = \left(\frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v} = -30{,}000\sin d\,\mathbf{v}$$

The magnitude of the force is $30{,}000\sin d$.

(b)

d	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Force	0	523.6	1047.0	1570.1	2092.7	2614.7	3135.9	3656.1	4175.2	4693.0	5209.4

(c) Force perpendicular to the hill when $d = 5°$:

Force $= \sqrt{(30{,}000)^2 - (2614.7)^2} \approx 29{,}885.8$ pounds

68. Force due to gravity: $\mathbf{F} = -5400\mathbf{j}$

Unit vector along hill: $\mathbf{v} = (\cos 10°)\mathbf{i} + (\sin 10°)\mathbf{j}$

Projection of $\mathbf{F}$ onto $\mathbf{v}$:

$$\begin{aligned}\mathbf{w}_1 &= \text{proj}_{\mathbf{v}}\mathbf{F}\\ &= \left(\frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\\ &= (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \text{ because } \mathbf{v} \text{ is a unit vector, } \|\mathbf{v}\| = 1\\ &= [(0)(\cos 10°) + (-5400)(\sin 10°)]\mathbf{v}\\ &= -5400(\sin 10°)\mathbf{v} = -937.7\mathbf{v}\end{aligned}$$

The magnitude of the force is 937.7, so a force of 937.7 pounds is required to keep the vehicle from rolling down the hill.

Force perpendicular to the hill: Force $= \sqrt{(5400)^2 - (937)^2} \approx 5318.0$ pounds

69. $\mathbf{w} = (245)(3) = 735$ newton-meters

70. work $= (2400)(5) = 12{,}000$ foot-pounds

71. $\mathbf{w} = (\cos 30°)(45)(20) \approx 779.4$ foot-pounds

72. work $= (\cos 35°)(15{,}691)(800)$
$\approx 10{,}282{,}652$ newton-meters

73. $\mathbf{w} = (\cos 30°)(250)(100) \approx 21{,}650.64$ foot-pounds

74. work $= (\cos\theta)\|\mathbf{F}\|\,\|\overrightarrow{PQ}\|$
$= (\cos 20°)(25 \text{ pounds})(50 \text{ feet})$
$= 1174.62$ foot-pounds

75. False. Work is represented by a scalar.

76. True.
$W = \mathbf{F} \cdot \overrightarrow{PQ} = 0$ if $\mathbf{F}$ and $\overrightarrow{PQ}$ are orthogonal.

77. (a) $\mathbf{u} \cdot \mathbf{v} = 0 \implies \mathbf{u}$ and $\mathbf{v}$ are orthogonal and $\theta = \dfrac{\pi}{2}$.

(b) $\mathbf{u} \cdot \mathbf{v} > 0 \implies \cos\theta > 0 \implies 0 \le \theta < \dfrac{\pi}{2}$

(c) $\mathbf{u} \cdot \mathbf{v} < 0 \implies \cos\theta < 0 \implies \dfrac{\pi}{2} < \theta \le \pi$

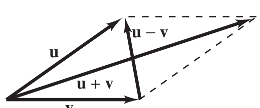
78. (a) $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u} \implies \mathbf{u}$ and $\mathbf{v}$ are parallel.

(b) $\text{proj}_{\mathbf{v}}\mathbf{u} = 0 \implies \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

79. In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

$$\begin{aligned}(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) &= (\mathbf{u} + \mathbf{v}) \cdot \mathbf{u} - (\mathbf{u} + \mathbf{v}) \cdot \mathbf{v}\\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v}\\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0\end{aligned}$$

Therefore, the diagonals are orthogonal.



80. Let $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$.

$$\begin{aligned}\mathbf{u} - \mathbf{v} &= \langle u_1 - v_1, u_2 - v_2\rangle\\ \|\mathbf{u} - \mathbf{v}\|^2 &= (u_1 - v_1)^2 + (u_2 - v_2)^2\\ &= u_1^2 - 2u_1v_1 + v_1^2 + u_2^2 - 2u_2v_2 + v_2^2\\ &= u_1^2 + u_2^2 + v_1^2 + v_2^2 - 2u_1v_1 - 2u_2v_2\\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(u_1v_1 + u_2v_2)\\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v}\end{aligned}$$

81. $$\begin{aligned}\sqrt{42} \cdot \sqrt{24} &= \sqrt{1008}\\ &= \sqrt{144 \cdot 7}\\ &= 12\sqrt{7}\end{aligned}$$

82. $$\begin{aligned}\sqrt{18} \cdot \sqrt{112} &= \sqrt{18 \cdot 112}\\ &= \sqrt{2 \cdot 3^2 \cdot 2^4 \cdot 7}\\ &= (3 \cdot 2^2)\sqrt{2 \cdot 7}\\ &= 12\sqrt{14}\end{aligned}$$

83. $$\begin{aligned}\sqrt{-3}\,\sqrt{-8} &= \left(\sqrt{3}i\right)\left(2\sqrt{2}i\right)\\ &= 2\sqrt{6}i^2\\ &= -2\sqrt{6}\end{aligned}$$

84. $$\begin{aligned}\sqrt{-12} \cdot \sqrt{-96} &= i\sqrt{12} \cdot i\sqrt{96}\\ &= i^2\sqrt{12 \cdot 96}\\ &= \sqrt{2^2 \cdot 3 \cdot 2^5 \cdot 3}\\ &= \sqrt{2 \cdot 2^6 \cdot 3^2}\\ &= (2^3 \cdot 3)\sqrt{2}\\ &= -24\sqrt{2}\end{aligned}$$

85. $$\begin{aligned}\sin 2x - \sqrt{3}\sin x &= 0\\ 2\sin x\cos x - \sqrt{3}\sin x &= 0\\ \sin x\left(2\cos x - \sqrt{3}\right) &= 0\end{aligned}$$

$\sin x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = 0, \pi$ $\qquad \cos x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{6}, \dfrac{11\pi}{6}$

86. $\sin 2x + \sqrt{2}\cos x = 0$

$$2\sin x\cos x + \sqrt{2}\cos x = 0$$
$$\cos x\left(2\sin x + \sqrt{2}\right) = 0$$
$$\cos x = 0 \qquad 2\sin x + \sqrt{2} = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = -\frac{\sqrt{2}}{2}$$
$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$
$$x = \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

87.
$$2\tan x = \tan 2x$$
$$2\tan x = \frac{2\tan x}{1 - \tan^2 x}$$
$$2\tan x(1 - \tan^2 x) = 2\tan x$$
$$2\tan x(1 - \tan^2 x) - 2\tan x = 0$$
$$2\tan x[(1 - \tan^2 x) - 1] = 0$$
$$2\tan x(-\tan^2 x) = 0$$
$$-2\tan^3 x = 0$$
$$\tan x = 0$$
$$x = 0, \pi$$

88. $\cos 2x - 3\sin x = 2$

$$1 - 2\sin^2 x - 3\sin x - 2 = 0$$
$$2\sin^2 x + 3\sin x + 1 = 0$$
$$(2\sin x + 1)(\sin x + 1) = 0$$
$$2\sin x + 1 = 0 \qquad \sin x + 1 = 0$$
$$\sin x = -\frac{1}{2} \qquad \sin x = -1$$
$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{3\pi}{2}$$
$$x = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

For Exercises 89–92:

$\sin u = -\frac{12}{13}$, **u in Quadrant IV** $\Rightarrow$ $\cos u = \frac{5}{13}$ $\qquad$ $\cos v = \frac{24}{25}$, **v in Quadrant IV** $\Rightarrow$ $\sin v = -\frac{7}{25}$

89. $\sin(u - v) = \sin u\cos v - \cos u\sin v$

$$= \left(-\tfrac{12}{13}\right)\left(\tfrac{24}{25}\right) - \left(\tfrac{5}{13}\right)\left(-\tfrac{7}{25}\right)$$
$$= -\tfrac{253}{325}$$

90. $\sin u = -\frac{12}{13}$, $\cos u = \sqrt{1 - \left(-\frac{12}{13}\right)^2} = \frac{5}{13}$

$\cos v = \frac{24}{25}$, $\sin v = -\sqrt{1 - \left(\frac{24}{25}\right)^2} = -\frac{7}{25}$

$$\sin(u + v) = \sin u\cos v + \cos u\sin v$$
$$= \left(-\tfrac{12}{13}\right)\left(\tfrac{24}{25}\right) + \left(\tfrac{5}{13}\right)\left(-\tfrac{7}{25}\right)$$
$$= -\tfrac{323}{325}$$

91. $\cos(v - u) = \cos v\cos u + \sin v\sin u$

$$= \left(\frac{24}{25}\right)\left(\frac{5}{13}\right) + \left(-\frac{7}{25}\right)\left(-\frac{12}{13}\right)$$
$$= \frac{204}{325}$$

92. $\sin u = -\dfrac{12}{13}$, $\cos u = \dfrac{5}{13}$, $\tan u = -\dfrac{12}{5}$

$\cos v = \dfrac{24}{25}$, $\sin v = -\dfrac{7}{25}$, $\tan v = -\dfrac{7}{24}$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u\tan v}$$
$$= \frac{\left(-\frac{12}{5}\right) - \left(-\frac{7}{24}\right)}{1 + \left(-\frac{12}{5}\right)\left(-\frac{7}{24}\right)} = \frac{-\frac{253}{120}}{\frac{17}{10}}$$
$$= -\frac{253}{204}$$

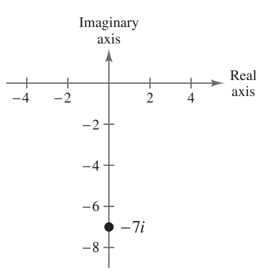
Section 6.5 Trigonometric Form of a Complex Number

- You should be able to graphically represent complex numbers and know the following facts about them.
- The absolute value of the complex number $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.
- The trigonometric form of the complex number $z = a + bi$ is $z = r(\cos\theta + i\sin\theta)$ where
  (a) $a = r\cos\theta$
  (b) $b = r\sin\theta$
  (c) $r = \sqrt{a^2 + b^2}$; r is called the modulus of z.
  (d) $\tan\theta = \dfrac{b}{a}$; θ is called the argument of z.
- Given $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$:
  (a) $z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
  (b) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)],\ z_2 \neq 0$
- You should know DeMoivre's Theorem: If $z = r(\cos\theta + i\sin\theta)$, then for any positive integer n,
  $z^n = r^n(\cos n\theta + i\sin n\theta)$.
- You should know that for any positive integer n, $z = r(\cos\theta + i\sin\theta)$ has n distinct nth roots given by
  $$\sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$$
  where $k = 0, 1, 2, \ldots, n - 1$.

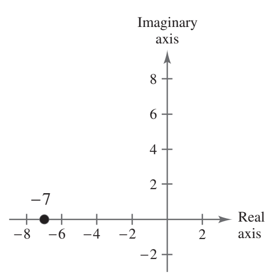
Vocabulary Check

1. absolute value
2. trigonometric form; modulus; argument
3. DeMoivre's
4. n^{th} root

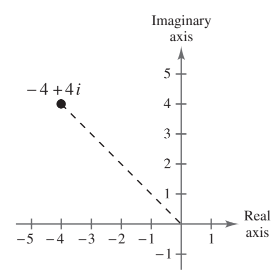
1. $|-7i| = \sqrt{0^2 + (-7)^2}$
$= \sqrt{49} = 7$



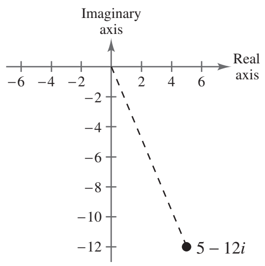
2. $|-7| = \sqrt{(-7)^2 + 0^2} = \sqrt{49} = 7$



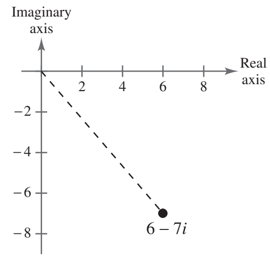
3. $|-4 + 4i| = \sqrt{(-4)^2 + (4)^2}$
$= \sqrt{32} = 4\sqrt{2}$



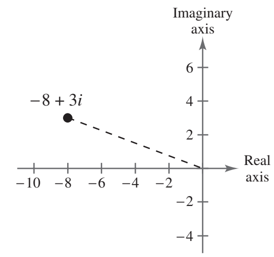
4. $|5 - 12i| = \sqrt{5^2 + (-12)^2}$
$= \sqrt{169} = 13$



5. $|6 - 7i| = \sqrt{6^2 + (-7)^2}$
$= \sqrt{85}$



6. $|-8 + 3i| = \sqrt{(-8)^2 + (3)^2}$
$= \sqrt{73}$



7. $z = 3i$

$r = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{3}{0}$, undefined $\Rightarrow\ \theta = \dfrac{\pi}{2}$

$z = 3\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

8. $z = -2$

$r = \sqrt{(-2)^2 + 0^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{0}{-2}\ \Rightarrow\ \theta = \pi$

$z = 2(\cos\pi + i\sin\pi)$

9. $z = 3 - i$

$r = \sqrt{(3)^2 + (-1)^2} = \sqrt{10}$

$\tan\theta = -\dfrac{1}{3}$, θ is in Quadrant IV.

$\theta \approx 5.96$ radians

$z \approx \sqrt{10}(\cos 5.96 + i\sin 5.96)$

10. $z = -1 + \sqrt{3}i$

$r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{\sqrt{3}}{-1} = -\sqrt{3}\ \Rightarrow\ \theta = \dfrac{2\pi}{3}$

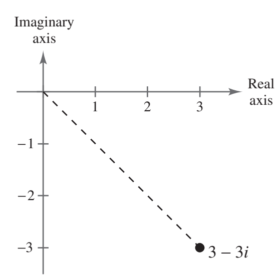
$z = 2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

11. $z = 3 - 3i$

$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \dfrac{-3}{3} = -1$, θ is in Quadrant IV $\Rightarrow\ \theta = \dfrac{7\pi}{4}$.

$z = 3\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$

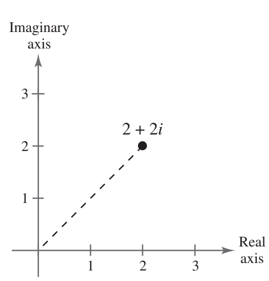


12. $z = 2 + 2i$

$r = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \dfrac{2}{2} = 1\ \Rightarrow\ \theta = \dfrac{\pi}{4}$

$z = 2\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

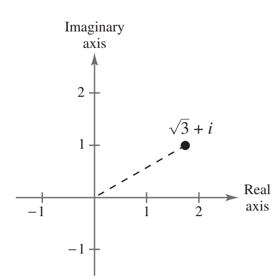


13. $z = \sqrt{3} + i$

$r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}\ \Rightarrow\ \theta = \dfrac{\pi}{6}$

$z = 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

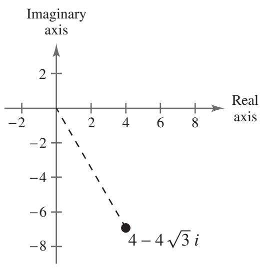


14. $z = 4 - 4\sqrt{3}i$

$r = \sqrt{4^2 + (-4\sqrt{3})^2} = 8$

$\tan\theta = \dfrac{-4\sqrt{3}}{4} = -\sqrt{3} \implies \theta = \dfrac{5\pi}{3}$

$z = 8\left(\cos\dfrac{5\pi}{3} + i\sin\dfrac{5\pi}{3}\right)$

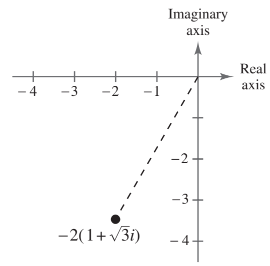


15. $z = -2(1 + \sqrt{3}i)$

$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{16} = 4$

$\tan\theta = \dfrac{\sqrt{3}}{1} = \sqrt{3}$, θ is in Quadrant III $\implies \theta = \dfrac{4\pi}{3}$.

$z = 4\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

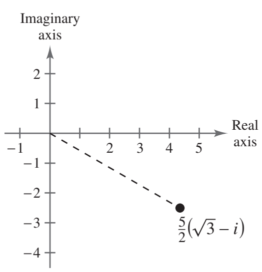


16. $z = \dfrac{5}{2}(\sqrt{3} - i)$

$r = \sqrt{\left(\dfrac{5}{2}\sqrt{3}\right)^2 + \left(\dfrac{5}{2}(-1)\right)^2} = \sqrt{\dfrac{100}{4}} = \sqrt{25} = 5$

$\tan\theta = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3} \implies \theta = \dfrac{11\pi}{6}$

$z = 5\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)$

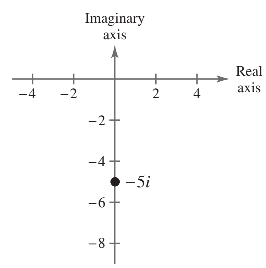


17. $z = -5i$

$r = \sqrt{0^2 + (-5)^2} = \sqrt{25} = 5$

$\tan\theta = \dfrac{-5}{0}$, undefined $\implies \theta = \dfrac{3\pi}{2}$

$z = 5\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$

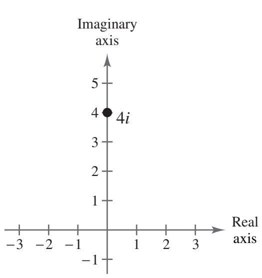


18. $z = 4i$

$r = \sqrt{0^2 + 4^2} = \sqrt{16^2} = 4$

$\tan\theta = \dfrac{4}{0}$, undefined $\implies \theta = \dfrac{\pi}{2}$

$z = 4\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

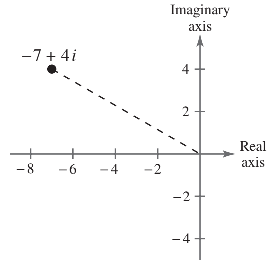


19. $z = -7 + 4i$

$r = \sqrt{(-7)^2 + (4)^2} = \sqrt{65}$

$\tan\theta = \dfrac{4}{-7}$, θ is in Quadrant II $\implies \theta \approx 2.62$.

$z \approx \sqrt{65}(\cos 2.62 + i\sin 2.62)$

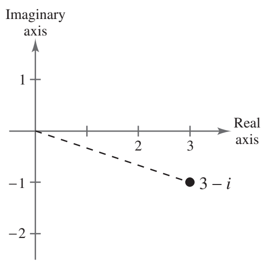


20. $z = 3 - i$

$r = \sqrt{(3)^2 + (-1)^2} = \sqrt{10}$

$\tan\theta = \dfrac{-1}{3} = \theta \approx 5.96$ radians

$z = \sqrt{10}\,(\cos 5.96 + i \sin 5.96)$

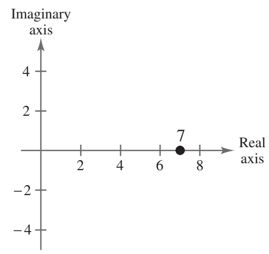


21. $z = 7 + 0i$

$r = \sqrt{(7)^2 + (0)^2} = \sqrt{49} = 7$

$\tan\theta = \dfrac{0}{7} = 0 \implies \theta = 0$

$z = 7(\cos 0 + i \sin 0)$

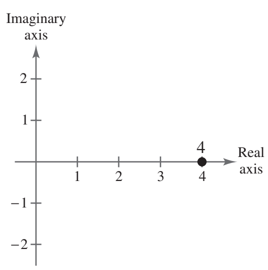


22. $z = 4$

$r = \sqrt{4^2 + 0^2} = \sqrt{16^2} = 4$

$\tan\theta = \dfrac{0}{4} = 0 \implies \theta = 0$

$z = 4(\cos 0 + i \sin 0)$



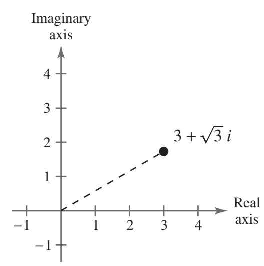
23. $z = 3 + \sqrt{3}i$

$r = \sqrt{(3)^2 + \left(\sqrt{3}\right)^2} = \sqrt{12}$

$= 2\sqrt{3}$

$\tan\theta = \dfrac{\sqrt{3}}{3} \implies \theta = \dfrac{\pi}{6}$

$z = 2\sqrt{3}\left(\cos\dfrac{\pi}{6} + i \sin\dfrac{\pi}{6}\right)$

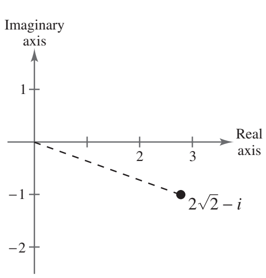


24. $z = 2\sqrt{2} - i$

$r = \sqrt{\left(2\sqrt{2}\right)^2 + (-1)^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{-1}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{4} \implies \theta \approx 5.94$ radians

$z = 3(\cos 5.94 + i \sin 5.94)$

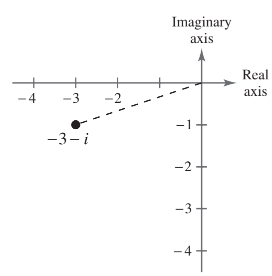


25. $z = -3 - i$

$r = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$

$\tan\theta = \dfrac{-1}{-3} = \dfrac{1}{3}$, θ is in Quadrant III $\implies \theta \approx 3.46.$

$z \approx \sqrt{10}\,(\cos 3.46 + i \sin 3.46)$

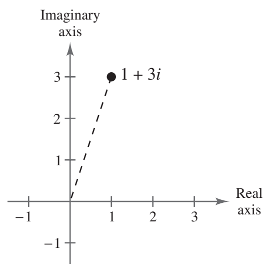


26. $z = 1 + 3i$

$r = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\tan\theta = \frac{3}{1} = 3 \implies \theta \approx 1.25$ radians

$z \approx \sqrt{10}\,(\cos 1.25 + i \sin 1.25)$



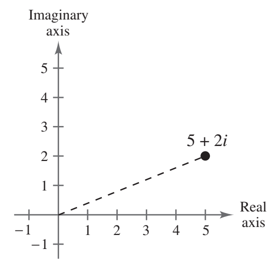
27. $z = 5 + 2i$

$r = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\tan\theta = \frac{2}{5}$

$\theta \approx 0.38$

$z \approx \sqrt{29}(\cos 0.38 + i \sin 0.38)$



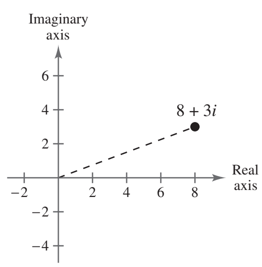
28. $z = 8 + 3i$

$r = \sqrt{8^2 + 3^2} = \sqrt{73}$

$\tan\theta = \dfrac{3}{8}$

$\theta = 0.36$

$z \approx \sqrt{73}(\cos 0.36 + i \sin 0.36)$



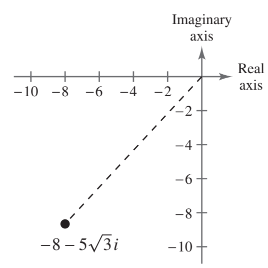
29. $z = -8 - 5\sqrt{3}i$

$r = \sqrt{(-8)^2 + \left(-5\sqrt{3}\right)^2} = \sqrt{139}$

$\tan\theta = \dfrac{5\sqrt{3}}{8}$

$\theta \approx 3.97$

$z \approx \sqrt{139}(\cos 3.97 + i \sin 3.97)$



30. $z = -9 - 2\sqrt{10}i$

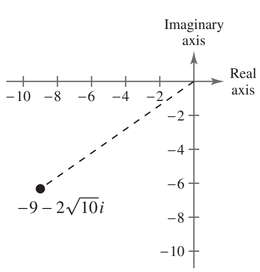
$r = \sqrt{(9)^2 + \left(-2\sqrt{10}\right)^2} = \sqrt{121}$

$r = 11$

$\tan\theta = \dfrac{-2\sqrt{10}}{9}$

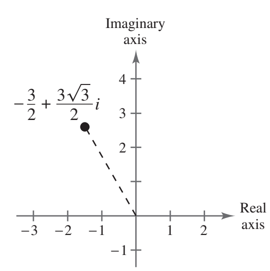
$\theta = 3.75$

$z \approx 11(\cos 3.75 + i \sin 3.75)$



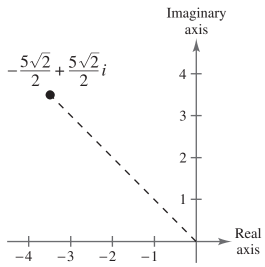
31. $3(\cos 120° + i \sin 120°) = 3\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)$

$= -\dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}i$



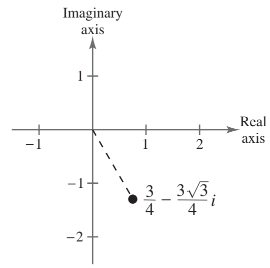
32. $5(\cos 135° + i \sin 135°) = 5\left[-\dfrac{\sqrt{2}}{2} + i\left(\dfrac{\sqrt{2}}{2}\right)\right]$

$= -\dfrac{5\sqrt{2}}{2} + \dfrac{5\sqrt{2}}{2}i$



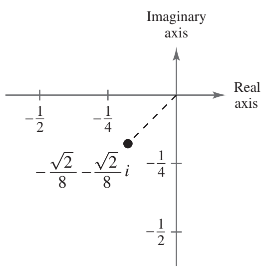
33. $\dfrac{3}{2}(\cos 300° + i \sin 300°) = \dfrac{3}{2}\left[\dfrac{1}{2} + i\left(-\dfrac{\sqrt{3}}{2}\right)\right]$

$= \dfrac{3}{4} - \dfrac{3\sqrt{3}}{4}i$

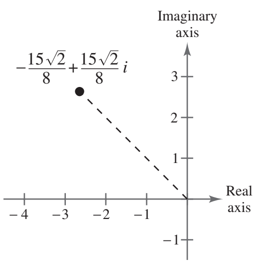


34. $\dfrac{1}{4}(\cos 225° + i \sin 225°) = \dfrac{1}{4}\left(-\dfrac{\sqrt{2}}{2} + i\left(-\dfrac{\sqrt{2}}{2}\right)\right)$

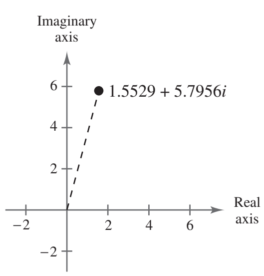
$= -\dfrac{\sqrt{2}}{8} - i\dfrac{\sqrt{2}}{8}$



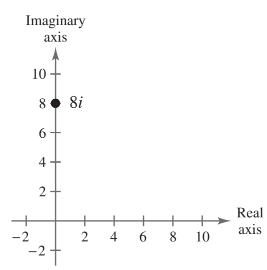
35. $3.75\left(\cos \dfrac{3\pi}{4} + i \sin \dfrac{3\pi}{4}\right) = -\dfrac{15\sqrt{2}}{8} + \dfrac{15\sqrt{2}}{8}i$



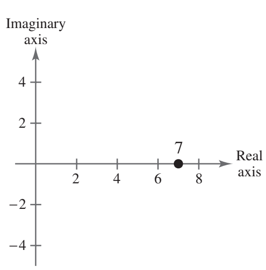
36. $6\left(\cos \dfrac{5\pi}{12} + i \sin \dfrac{5\pi}{12}\right) \approx 1.5529 + 5.7956i$



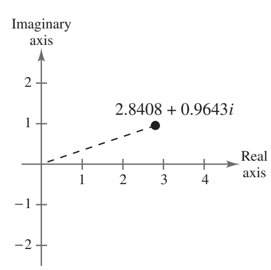
37. $8\left(\cos \dfrac{\pi}{2} + i \sin \dfrac{\pi}{2}\right) = 8(0 + i) = 8i$



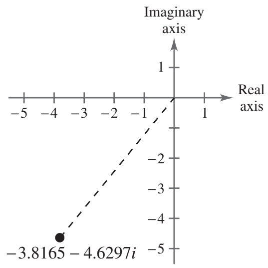
38. $7(\cos 0° + i \sin 0°) = 7$



39. $3[\cos(18°45') + i \sin(18°45')] \approx 2.8408 + 0.9643i$



40. $6[\cos(230°\ 30') + i\sin(230°\ 30')] \approx -3.8165 - 4.6297i$



41. $5\left(\cos\dfrac{\pi}{9} + i\sin\dfrac{\pi}{9}\right) \approx 4.6985 + 1.7101i$

42. $10\left(\cos\dfrac{2\pi}{5} + i\sin\dfrac{2\pi}{5}\right) \approx 3.0902 + 9.5106i$

43. $3(\cos 165.5° + i\sin 165.5°) \approx -2.9044 + 0.7511i$

44. $9(\cos 58° + i\sin 58°) \approx 4.7693 + 7.6324i$

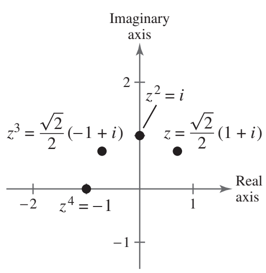
45. $z = \dfrac{\sqrt{2}}{2}(1 + i) = \cos 45° + i\sin 45°$

$z^2 = \cos 90° + i\sin 90° = i$

$z^3 = \cos 135° + i\sin 135° = \dfrac{\sqrt{2}}{2}(-1 + i)$

$z^4 = \cos 180° + i\sin 180° = -1$

The absolute value of each is 1, and consecutive powers of z are each 45° apart.



46. $z = \dfrac{1}{2}\left(1 + \sqrt{3}i\right)$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

$r = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2} = 1$

$\tan\theta = \sqrt{3}$

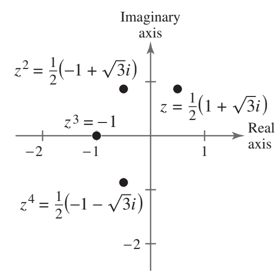
$\theta = \dfrac{\pi}{3}$

$z = 1\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right) = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$z^2 = 1^2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right) = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$z^3 = 1^3(\cos\pi + i\sin\pi) = -1$

$z^4 = 1^4\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right) = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$



The absolute value of each is 1 and consecutive powers of z are each $\pi/3$ radians apart.

47. $$\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right] = (2)(6)\left[\cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right)\right]$$
$$= 12\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

48. $$\left[\frac{3}{4}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right] = \left(\frac{3}{4}\right)(4)\left[\cos\left(\frac{\pi}{3} + \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)\right]$$
$$= 3\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$$

49. $$\left[\tfrac{5}{3}(\cos 140° + i\sin 140°)\right]\left[\tfrac{2}{3}(\cos 60° + i\sin 60°)\right] = \left(\tfrac{5}{3}\right)\left(\tfrac{2}{3}\right)[\cos(140° + 60°) + i\sin(140° + 60°)]$$
$$= \tfrac{10}{9}(\cos 200° + i\sin 200°)$$

50. $[0.5(\cos 100° + i \sin 100°)][0.8(\cos 300° + i \sin 300°)] = (0.5)(0.8)[\cos(100° + 300°) + i \sin(100° + 300°)]$
$= 0.4(\cos 400° + i \sin 400°)$
$= 0.4(\cos 40° + i \sin 40°)$

51. $[0.45(\cos 310° + i \sin 310°)][0.60(\cos 200° + i \sin 200°)] = (0.45)(0.60)[\cos(310° + 200°) + i \sin(310° + 200°)]$
$= 0.27(\cos 510° + i \sin 510°)$
$= 0.27(\cos 150° + i \sin 150°)$

52. $(\cos 5° + i \sin 5°)(\cos 20° + i \sin 20°) = \cos(5° + 20°) + i \sin(5° + 20°) = \cos 25° + i \sin 25°$

53. $\dfrac{\cos 50° + i \sin 50°}{\cos 20° + i \sin 20°} = \cos(50° - 20°) + i \sin(50° - 20°) = \cos 30° + i \sin 30°$

54. $\dfrac{2(\cos 120° + i \sin 120°)}{4(\cos 40° + i \sin 40°)} = \dfrac{2}{4}[\cos(120° - 40°) + i \sin(120° - 40°)]$
$= \dfrac{1}{2}(\cos 80° + i \sin 80°)$

55. $\dfrac{\cos\frac{5\pi}{3} + i \sin\frac{5\pi}{3}}{\cos \pi + i \sin \pi} = \cos\left(\dfrac{5\pi}{3} - \pi\right) + i \sin\left(\dfrac{5\pi}{3} - \pi\right) = \cos\left(\dfrac{2\pi}{3}\right) + i \sin\left(\dfrac{2\pi}{3}\right)$

56. $\dfrac{5[\cos (4.3) + i \sin(4.3)]}{4[\cos (2.1) + i \sin(2.1)]} = \dfrac{5}{4}[\cos(4.3 - 2.1) + i \sin(4.3 - 2.1)$
$= \dfrac{5}{4}[\cos(2.2) + i \sin(2.2)]$

57. $\dfrac{12(\cos 52° + i \sin 52°)}{3(\cos 110° + i \sin 110°)} = 4[\cos(52° - 110°) + i \sin(52° - 110°)]$
$= 4[\cos(-58°) + i \sin(-58°)]$
$= 4(\cos 302° + i \sin 302°)$

58. $\dfrac{6[\cos 40° + i \sin 40°]}{7[\cos 100° + i \sin 100°]} = \dfrac{6}{7}[\cos(40° - 100°) + i \sin(40° - 100°)]$
$= \dfrac{6}{7}[\cos 300° + i \sin 300°]$

59. (a) $2 + 2i = 2\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)$

$1 - i = \sqrt{2}\left[\cos\left(-\dfrac{\pi}{4}\right) + i \sin\left(-\dfrac{\pi}{4}\right)\right] = \sqrt{2}\left(\cos \dfrac{7\pi}{4} + i \sin \dfrac{7\pi}{4}\right)$

(b) $(2 + 2i)(1 - i) = \left[2\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)\right]\left[\sqrt{2}\left(\cos\left(\dfrac{7\pi}{4}\right) + i \sin\left(\dfrac{7\pi}{4}\right)\right)\right] = 4(\cos 2\pi + i \sin 2\pi)$

$= 4(\cos 0 + i \sin 0) = 4$

(c) $(2 + 2i)(1 - i) = 2 - 2i + 2i - 2i^2 = 2 + 2 = 4$

60. (a) $\sqrt{3} + i = 2(\cos 30° + i \sin 30°)$

$1 + i = \sqrt{2}(\cos 45° + i \sin 45°)$

(b) $(\sqrt{3} + i)(1 + i) = [2(\cos 30° + i \sin 30°)][\sqrt{2}(\cos 45° + i \sin 45°)]$

$= 2\sqrt{2}(\cos 75° + i \sin 75°)$

$= 2\sqrt{2}\left[\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) + \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right)i\right]$

$= (\sqrt{3} - 1) + (\sqrt{3} + 1)i \approx 0.732 + 2.732i$

(c) $(\sqrt{3} + i)(1 + i) = \sqrt{3} + (\sqrt{3} + 1)i + i^2 = (\sqrt{3} - 1) + (\sqrt{3} + 1)i \approx 0.732 + 2.732i$

61. (a) $-2i = 2\left[\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)\right] = 2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

$1 + i = \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

(b) $-2i(1 + i) = 2\left[\cos\left(\frac{3\pi}{2}\right) + i \sin\left(\frac{3\pi}{2}\right)\right]\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]$

$= 2\sqrt{2}\left[\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right)\right]$

$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right] = 2 - 2i$

(c) $-2i(1 + i) = -2i - 2i^2 = -2i + 2 = 2 - 2i$

62. (a) $4 = 4(\cos 0 + i \sin 0)$

$1 - \sqrt{3}i = 2\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(\frac{-\pi}{3}\right)\right)$

(b) $4(1 - \sqrt{3}i) = 8\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right)$

$= 8\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 4 - 4\sqrt{3}i$

(c) $4(1 - \sqrt{3}i) = 4 - 4\sqrt{3}i$

63. (a) $3 + 4i \approx 5(\cos 0.93 + i \sin 0.93)$

$1 - \sqrt{3}i = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$

(b) $\frac{3 + 4i}{1 - \sqrt{3}i} \approx \frac{5(\cos 0.93 + i \sin 0.93)}{2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)}$

$\approx 2.5[\cos(-4.31) + i \sin(-4.31)]$

$= \frac{5}{2}(\cos 1.97 + i \sin 1.97)$

$\approx -0.982 + 2.299i$

(c) $\frac{3 + 4i}{1 - \sqrt{3}i} = \frac{3 + 4i}{1 - \sqrt{3}i} \cdot \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i}$

$= \frac{3 + (4 + 3\sqrt{3})i + 4\sqrt{3}i^2}{1 + 3}$

$= \frac{3 - 4\sqrt{3}}{4} + \frac{4 + 3\sqrt{3}}{4}i$

$\approx -0.982 + 2.299i$

64. (a) $1 + \sqrt{3}i = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

$6 - 3i \approx 3\sqrt{5}[\cos(-0.464) + i \sin(-0.464)]$

(b) $\frac{1 + \sqrt{3}i}{6 - 3i} \approx \frac{2}{3\sqrt{5}}\left[\cos\left(\frac{\pi}{3} + 0.464\right) + i \sin\left(\frac{\pi}{3} + 0.464\right)\right] \approx \frac{2\sqrt{5}}{15}[\cos 1.51 + i \sin 1.51] \approx 0.018 + 0.298i$

(c) $\frac{1 + \sqrt{3}i}{6 - 3i} \cdot \frac{6 + 3i}{6 + 3i} = \frac{(6 - 3\sqrt{3}) + i(3 + 6\sqrt{3})}{45} = \frac{2 - \sqrt{3}}{15} + i\,\frac{1 + 2\sqrt{3}}{15} \approx 0.018 + 0.298i$

65. (a) $5 = 5(\cos 0 + i \sin 0)$

$2 + 3i \approx \sqrt{13}(\cos 0.98 + i \sin 0.98)$

(b) $\dfrac{5}{2 + 3i} \approx \dfrac{5(\cos 0 + i \sin 0)}{\sqrt{13}(\cos 0.98 + i \sin 0.98)} = \dfrac{5}{\sqrt{13}}[\cos(-0.98) + i \sin(-0.98)] = \dfrac{5}{\sqrt{13}}(\cos 5.30 + i \sin 5.30) \approx 0.769 - 1.154i$

(c) $\dfrac{5}{2 + 3i} = \dfrac{5}{2 + 3i} \cdot \dfrac{2 - 3i}{2 - 3i} = \dfrac{10 - 15i}{13} = \dfrac{10}{13} - \dfrac{15}{13}i \approx 0.769 - 1.154i$

66. (a) $4i = 4(\cos 90° + i \sin 90°)$

$-4 + 2i = 2\sqrt{5}(\cos 153.4° + i \sin 153.4°)$

(b) $\dfrac{4i}{-4 + 2i} = \dfrac{4(\cos 90° + i \sin 90°)}{2\sqrt{5}(\cos 153.4° + i \sin 153.4°)}$

$= \dfrac{2\sqrt{5}}{5}(\cos 296.6° + i \sin 296.6°)$

$\approx 0.400 - 0.800i$

(c) $\dfrac{4i}{-4 + 2i} = \dfrac{4i}{-4 + 2i} \cdot \dfrac{-4 - i}{-4 - 2i}$

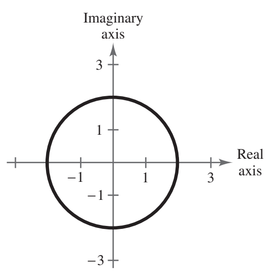
$= \dfrac{8 - 16i}{20} = \dfrac{2}{5} - \dfrac{4}{5}i = 0.400 - 0.800i$

67. Let $z = x + iy$ such that:

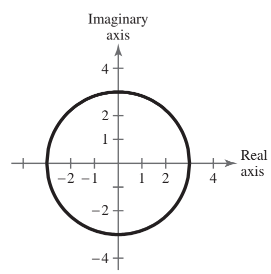
$|z| = 2 \implies 2 = \sqrt{x^2 + y^2}$

$\implies 4 = x^2 + y^2$:

circle with radius of 2

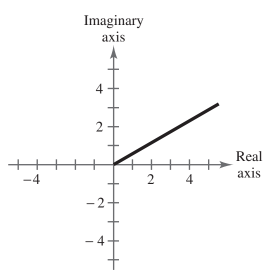


68. $|z| = 3$



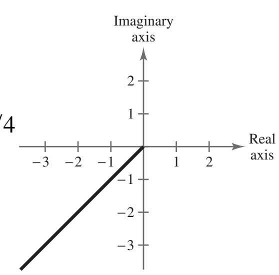
69. Let $\theta = \dfrac{\pi}{6}$.

Since $r \ge 0$, we have the portion of the line $\theta = \pi/6$ in Quadrant I.



70. $\theta = \dfrac{5\pi}{4}$

Since $r \ge 0$, we have the portion of the line $\theta = 5\pi/4$ in Quadrant III.



71. $(1 + i)^5 = \left[\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)\right]^5$

$= \left(\sqrt{2}\right)^5\left(\cos \dfrac{5\pi}{4} + i \sin \dfrac{5\pi}{4}\right)$

$= 4\sqrt{2}\left(-\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i\right)$

$= -4 - 4i$

72. $(2 + 2i)^6 = \left[2\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)\right]^6$

$= \left(2\sqrt{2}\right)^6\left(\cos \dfrac{6\pi}{4} + i \sin \dfrac{6\pi}{4}\right)$

$= 512\left(\cos \dfrac{3\pi}{2} + i \sin \dfrac{3\pi}{2}\right)$

$= -512i$

73. $(-1+i)^{10} = \left[\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^{10} = \left(\sqrt{2}\right)^{10}\left(\cos\frac{30\pi}{4} + i\sin\frac{30\pi}{4}\right)$

$= 32\left[\cos\left(\frac{3\pi}{2} + 6\pi\right) + i\sin\left(\frac{3\pi}{2} + 6\pi\right)\right] = 32\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$= 32[0 + i(-1)] = -32i$

74. $(3-2i)^8 = \left[\sqrt{13}\left(\cos\left(-\arctan\left(\frac{2}{3}\right)\right) + i\sin\left(-\arctan\left(\frac{2}{3}\right)\right)\right)\right]^8$

$= \left(\sqrt{13}\right)^8\left[\cos\left(-8\arctan\left(\frac{2}{3}\right)\right) + i\sin\left(-8\arctan\left(\frac{2}{3}\right)\right)\right]$

$= -239 + 28{,}560i$

75. $2\left(\sqrt{3} + i\right)^7 = 2\left[2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^7$

$= 2\left[2^7\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)\right]$

$= 256\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= -128\sqrt{3} - 128i$

76. $4\left(1 - \sqrt{3}i\right)^3 = 4\left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^3$

$= 4[2^3(\cos 5\pi + i\sin 5\pi)]$

$= 32(-1)$

$= -32$

77. $[5(\cos 20^\circ + i\sin 20^\circ)]^3 = 5^3(\cos 60^\circ + i\sin 60^\circ) = \frac{125}{2} + \frac{125\sqrt{3}}{2}i$

78. $[3(\cos 150^\circ + i\sin 150^\circ)]^4 = 3^4(\cos 600^\circ + i\sin 600^\circ)$

$= 81(\cos 240^\circ + i\sin 240^\circ)$

$= 81(-\cos 60^\circ - i\sin 60^\circ)$

$= -\frac{81}{2} - \frac{81\sqrt{3}}{2}i$

79. $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{12} = \cos\frac{12\pi}{4} + i\sin\frac{12\pi}{4}$

$= \cos 3\pi + i\sin 3\pi$

$= -1$

80. $\left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^8 = 2^8(\cos 4\pi + i\sin 4\pi)$

$= 256(\cos 0 + i\sin 0)$

$= 256$

81. $[5(\cos 3.2 + i\sin 3.2)]^4 = 5^4(\cos 12.8 + i\sin 12.8)$

$\approx 608.02 + 144.69i$

82. $(\cos 0 + i\sin 0)^{20} = \cos 0 + i\sin 0$

$= 1$

83. $(3-2i)^5 \approx [3.6056[\cos(-0.588) + i\sin(-0.588)]]^5$

$\approx (3.6056)^5[\cos(-2.94) + i\sin(-2.94)]$

$\approx -597 - 122i$

84. $\left(\sqrt{5} - 4i\right)^3 \approx \left[\sqrt{21}(\cos(-1.06106) + i\sin(-1.06106)\right]^3$

$\approx \left(\sqrt{21}\right)^3(\cos[(3)(-1.06106)] + i\sin[(3)(-1.06106)])$

$\approx -96.15 + 4.00i$

85. $[3(\cos 15^\circ + i\sin 15^\circ)]^4 = 81(\cos 60^\circ + i\sin 60^\circ)$

$= \frac{81}{2} + \frac{81\sqrt{3}}{2}i$

86. $[2(\cos 10^\circ + i\sin 10^\circ)]^8 = 256(\cos 80^\circ + i\sin 80^\circ)$

$\approx 44.45 + 252.11i$

87. $\left[2\left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}\right)\right]^5 = 2^5\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$
$= 32i$

88. $\left[2\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^6 = 64\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
$= -32\sqrt{2} + 32\sqrt{2}i$

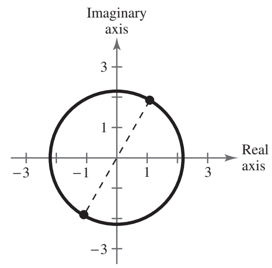
89. (a) Square roots of $5(\cos 120° + i \sin 120°)$:

$$\sqrt{5}\left[\cos\left(\frac{120° + 360°k}{2}\right) + i \sin\left(\frac{120° + 360°k}{2}\right)\right],\ k = 0,\ 1$$

$k = 0$: $\sqrt{5}(\cos 60° + i \sin 60°)$

$k = 1$: $\sqrt{5}(\cos 240° + i \sin 240°)$

(b)



(c) $\frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2}i,\ -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2}i$

90. (a) Square roots of $16(\cos 60° + i \sin 60°)$:

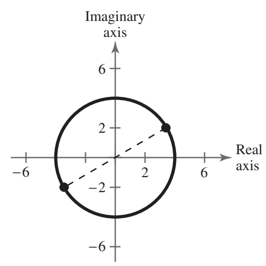
$$\sqrt{16}\left[\cos\left(\frac{60° + k\,360°}{2}\right) + i \sin\left(\frac{60° + k\,360°}{2}\right)\right], k = 0, 1$$

$k = 0$: $4(\cos 30° + i \sin 30°)$

$k = 1$: $4(\cos 210° + i \sin 210°)$

(c) $2\sqrt{3} + 2i, -2\sqrt{3} - 2i$

(b)



91. (a) Cube roots of $8\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$:

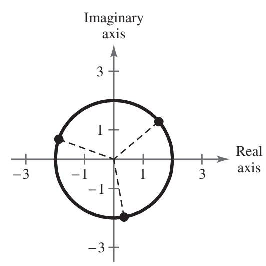
$$\sqrt[3]{8}\left[\cos\left(\frac{(2\pi/3) + 2\pi k}{3}\right) + i \sin\left(\frac{(2\pi/3) + 2\pi k}{3}\right)\right], k = 0, 1, 2$$

$k = 0$: $2\left(\cos \frac{2\pi}{9} + i \sin \frac{2\pi}{9}\right)$

$k = 1$: $2\left(\cos \frac{8\pi}{9} + i \sin \frac{8\pi}{9}\right)$

$k = 2$: $2\left(\cos \frac{14\pi}{9} + i \sin \frac{14\pi}{9}\right)$

(b)



(c) $1.5321 + 1.2856i, -1.8794 + 0.6840i, 0.3473 - 1.9696i$

92. (a) Fifth roots of $32\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$:

$$\sqrt[5]{32}\left[\cos\left(\frac{(5\pi/6) + 2k\pi}{5}\right) + i \sin\left(\frac{(5\pi/6) + 2k\pi}{5}\right)\right], k = 0, 1, 2, 3, 4$$

$k = 0$: $2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

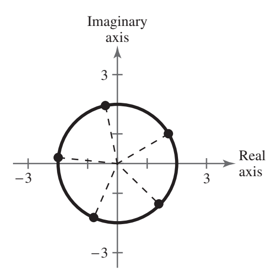
$k = 1$: $2\left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30}\right)$

$k = 2$: $2\left(\cos \frac{29\pi}{30} + i \sin \frac{29\pi}{30}\right)$

$k = 3$: $2\left(\cos \frac{41\pi}{30} + i \sin \frac{41\pi}{30}\right)$

$k = 4$: $2\left(\cos \frac{53\pi}{30} + i \sin \frac{53\pi}{30}\right)$

(b)



(c) $\sqrt{3} + i, -0.4158 + 1.9563i,$
$-1.9890 + 0.2091i, -0.8135 - 1.8271i,$
$1.4863 - 1.3383i$

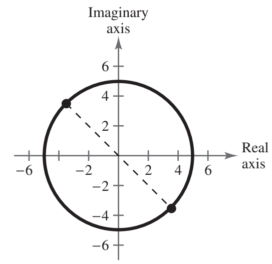
93. (a) Square roots of $-25i = 25\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$:

$$\sqrt{25}\left[\cos\left(\frac{\frac{3\pi}{2} + 2k\pi}{2}\right) + i \sin\left(\frac{\frac{3\pi}{2} + 2k\pi}{2}\right)\right],\ k = 0, 1$$

$k = 0$: $5\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

$k = 1$: $5\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

(b)



(c) $-\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i,\ \frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i$

94. (a) Fourth roots of $625i = 625\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

$$\sqrt[4]{625}\left[\cos\left(\frac{\frac{\pi}{2} + 2k\pi}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2k\pi}{2}\right)\right]$$

$k = 0, 1, 2, 3$

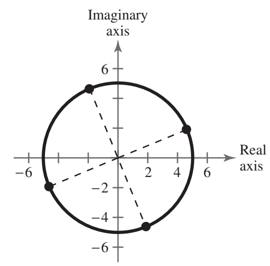
$k = 0$: $5\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)$

$k = 1$: $5\left(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}\right)$

$k = 2$: $5\left(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}\right)$

$k = 3$: $5\left(\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}\right)$

(b)



(c) $4.6194 + 1.9134i,\ -1.9134 + 4.6194i,$
$-4.6194 - 1.9134i,\ 1.9134 - 4.6194i$

95. (a) Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i) = 125\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$:

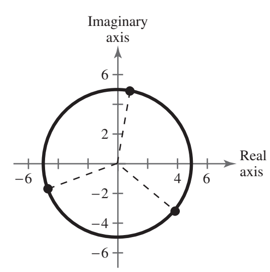
$$\sqrt[3]{125}\left[\cos\left(\frac{\frac{4\pi}{3} + 2k\pi}{3}\right) + i \sin\left(\frac{\frac{4\pi}{3} + 2k\pi}{3}\right)\right],\ k = 0, 1, 2$$

$k = 0$: $5\left(\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9}\right)$

$k = 1$: $5\left(\cos \frac{10\pi}{9} + i \sin \frac{10\pi}{9}\right)$

$k = 2$: $5\left(\cos \frac{16\pi}{9} + i \sin \frac{16\pi}{9}\right)$

(b)



(c) $0.8682 + 4.9240i,\ -4.6985 - 1.7101i,\ 3.8302 - 3.2140i$

96. (a) Cube roots of $-4\sqrt{2}(1 - i) = 8\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$:

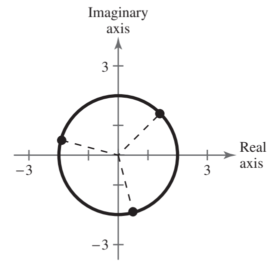
$$\sqrt[3]{8}\left[\cos\left(\frac{\frac{3\pi}{4} + 2k\pi}{3}\right) + i \sin\left(\frac{\frac{3\pi}{4} + 2k\pi}{3}\right)\right], \ k = 0, 1, 2$$

$k = 0$: $2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

$k = 1$: $2\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right)$

$k = 2$: $2\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right)$

(b)



(c) $\sqrt{2} + \sqrt{2}i, -1.9319 + 0.5176i, 0.5176 - 1.9319i$

97. (a) Fourth roots of $16 = 16(\cos 0 + i \sin 0)$:

$$\sqrt[4]{16}\left[\cos \frac{0 + 2\pi k}{4} + i \sin \frac{0 + 2\pi k}{4}\right], k = 0, 1, 2, 3$$

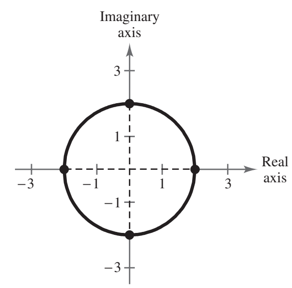
$k = 0$: $2(\cos 0 + i \sin 0)$

$k = 1$: $2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

$k = 2$: $2(\cos \pi + i \sin \pi)$

$k = 3$: $2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

(b)



(c) $2, 2i, -2, -2i$

98. (a) Fourth roots of $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$:

$$\sqrt[4]{1}\left[\cos\left(\frac{\frac{\pi}{2} + 2k\pi}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2k\pi}{4}\right)\right], \ k = 0, 1, 2, 3$$

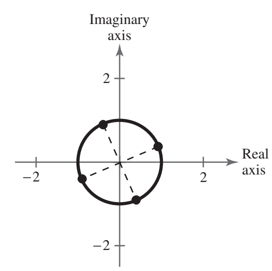
$k = 0$: $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$

$k = 1$: $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$

$k = 2$: $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$

$k = 3$: $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$

(b)



(c) $0.9239 + 0.3827i, -0.3827 + 0.9239i,$
$-0.9239 - 0.3827i, 0.3827 - 0.9239i$

99. (a) Fifth roots of $1 = \cos 0 + i \sin 0$:

$\cos\left(\frac{2k\pi}{5}\right) + i \sin\left(\frac{2k\pi}{5}\right), k = 0, 1, 2, 3, 4$

$k = 0$: $\cos 0 + i \sin 0$

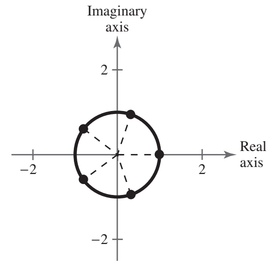
$k = 1$: $\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$

$k = 2$: $\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$

$k = 3$: $\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$

$k = 4$: $\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$

(b)



(c) $1, 0.3090 + 0.9511i, -0.8090 + 0.5878i, -0.8090 - 0.5878i, 0.3090 - 0.9511i$

100. (a) Cube roots of $1000 = 1000(\cos 0 + i \sin 0)$:

$\sqrt[3]{1000}\left(\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}\right)$

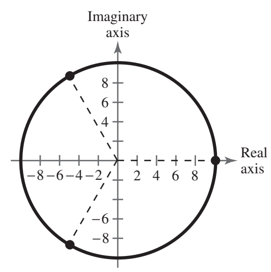
$k = 0, 1, 2$

$k = 0$: $10(\cos 0 + i \sin 0)$

$k = 1$: $10\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

$k = 2$: $10\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

(b)



(c) $10, -5 + 5\sqrt{3}i, -5 - 5\sqrt{3}i$

101. (a) Cube roots of $-125 = 125(\cos \pi + i \sin \pi)$:

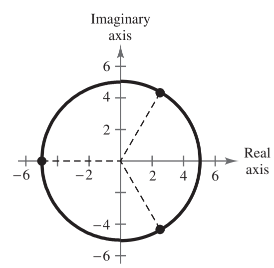
$\sqrt[3]{125}\left[\cos\left(\frac{\pi + 2\pi k}{3}\right) + i \sin\left(\frac{\pi + 2\pi k}{3}\right)\right], k = 0, 1, 2$

$k = 0$: $5\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

$k = 1$: $5(\cos \pi + i \sin \pi)$

$k = 2$: $5\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$

(b)



(c) $\frac{5}{2} + \frac{5\sqrt{3}}{2}i, -5, \frac{5}{2} - \frac{5\sqrt{3}}{2}i$

102. (a) Fourth roots of $-4 = 4(\cos \pi + i \sin \pi)$:

$$\sqrt[4]{4}\left[\cos\left(\frac{\pi + 2k\pi}{4}\right) + i \sin\left(\frac{\pi + 2k\pi}{4}\right)\right]$$

$k = 0, 1, 2, 3$

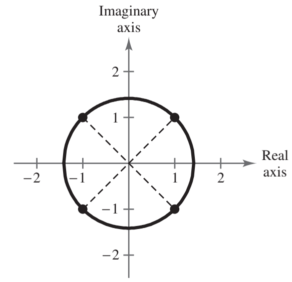
$k = 0$: $\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

$k = 1$: $\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

$k = 2$: $\sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$

$k = 3$: $\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

(b)



(c) $1 + i, -1 + i, -1 - i, 1 - i$

103. (a) Fifth roots of $128(-1 + i) = 128\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = 2^{15/2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$

$$2^{3/2}\left[\cos\left(\frac{\frac{3\pi}{4} + 2\pi k}{5}\right) + i \sin\left(\frac{\frac{3\pi}{4} + 2\pi k}{5}\right)\right], \ k = 0, 1, 2, 3, 4$$

$k = 0$: $2\sqrt{2}\left(\cos \frac{3\pi}{20} + i \sin \frac{3\pi}{20}\right)$

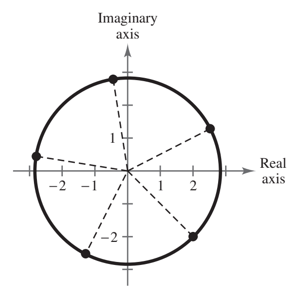
$k = 1$: $2\sqrt{2}\left(\cos \frac{11\pi}{20} + i \sin \frac{11\pi}{20}\right)$

$k = 2$: $2\sqrt{2}\left(\cos \frac{19\pi}{20} + i \sin \frac{19\pi}{20}\right)$

$k = 3$: $2\sqrt{2}\left(\cos \frac{27\pi}{20} + i \sin \frac{27\pi}{20}\right)$

$k = 4$: $2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

(b)



(c) $2.5201 + 1.2841i, -0.4425 + 2.7936i,$
$-2.7936 + 0.4425i, -1.2841 - 2.5201i, 2 - 2i$

104. (a) Sixth roots of $64i = 64\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

$$\sqrt[6]{64}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{6}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{6}\right)\right]$$

$k = 0, 1, 2, 3, 4, 5$

$k = 0$: $2\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$

$k = 1$: $2\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)$

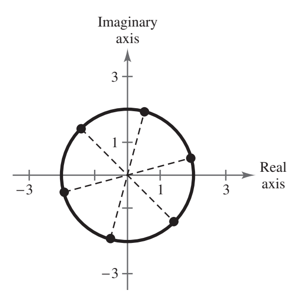
$k = 2$: $2\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

$k = 3$: $2\left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}\right)$

$k = 4$: $2\left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12}\right)$

$k = 5$: $2\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

(b)



(c) $1.9319 + 0.5176i, 0.5176 + 1.9319i, -\sqrt{2} + \sqrt{2}i,$
$-1.9319 - 0.5176i, -0.5176 - 1.9319i, \sqrt{2} - \sqrt{2}i$

105. $x^4 + i = 0$

$x^4 = -i$

The solutions are the fourth roots of $i = \cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}$:

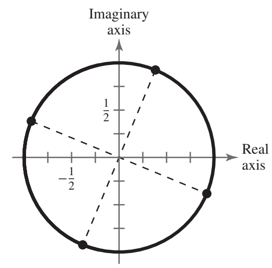
$$\sqrt[4]{1}\left[\cos\left(\frac{\frac{3\pi}{2} + 2k\pi}{4}\right) + i\sin\left(\frac{\frac{3\pi}{2} + 2k\pi}{4}\right)\right],\ k = 0, 1, 2, 3$$

$k = 0$: $\cos\dfrac{3\pi}{8} + i\sin\dfrac{3\pi}{8} \approx 0.3827 + 0.9239i$

$k = 1$: $\cos\dfrac{7\pi}{8} + i\sin\dfrac{7\pi}{8} \approx -0.9239 + 0.3827i$

$k = 2$: $\cos\dfrac{11\pi}{8} + i\sin\dfrac{11\pi}{8} \approx -0.3827 - 0.9239i$

$k = 3$: $\cos\dfrac{15\pi}{8} + i\sin\dfrac{15\pi}{8} \approx 0.9239 - 0.3827i$



106. $x^3 + 1 = 0$

$x^3 = -1$

The solutions are the cube roots of $-1 = \cos\pi + i\sin\pi$:

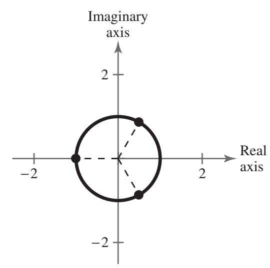
$$\cos\left(\frac{\pi + 2k\pi}{3}\right) + i\sin\left(\frac{\pi + 2k\pi}{3}\right)$$

$k = 0, 1, 2$

$k = 0$: $\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$k = 1$: $\cos\pi + i\sin\pi = -1$

$k = 2$: $\cos\dfrac{5\pi}{3} + i\sin\dfrac{5\pi}{3} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$



107. $x^5 + 243 = 0$

$x^5 = -243$

The solutions are the fifth roots of $-243 = 243(\cos\pi + i\sin\pi)$:

$$\sqrt[5]{243}\left[\cos\left(\frac{\pi + 2k\pi}{5}\right) + i\sin\left(\frac{\pi + 2k\pi}{5}\right)\right],\ k = 0, 1, 2, 3, 4$$

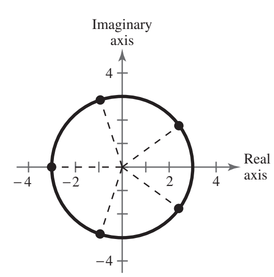
$k = 0$: $3\left(\cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}\right) \approx 2.4271 + 1.7634i$

$k = 1$: $3\left(\cos\dfrac{3\pi}{5} + i\sin\dfrac{3\pi}{5}\right) \approx -0.9271 + 2.8532i$

$k = 2$: $3(\cos\pi + i\sin\pi) = -3$

$k = 3$: $3\left(\cos\dfrac{7\pi}{5} + i\sin\dfrac{7\pi}{5}\right) \approx -0.9271 - 2.8532i$

$k = 4$: $3\left(\cos\dfrac{9\pi}{5} + i\sin\dfrac{9\pi}{5}\right) \approx 2.4271 - 1.7634i$



108. $x^3 - 27 = 0$

$x^3 = 27$

The solutions are the cube roots of $27 = 27(\cos 0 + i \sin 0)$:

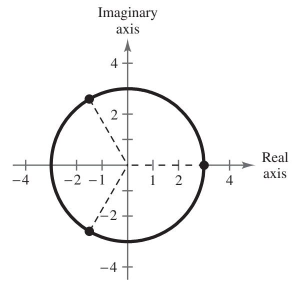
$\sqrt[3]{27}\left[\cos\left(\frac{2k\pi}{3}\right) + i \sin\left(\frac{2k\pi}{3}\right)\right]$

$k = 0, 1, 2$

$k = 0$: $3(\cos 0 + i \sin 0) = 3$

$k = 1$: $3\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$k = 2$: $3\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$



109. $x^4 + 16i = 0$

$x^4 = -16i$

The solutions are the fourth roots of $-16i = 16\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$:

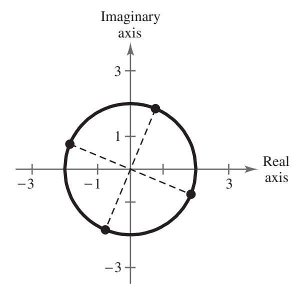
$\sqrt[4]{16}\left[\cos \frac{\frac{3\pi}{2} + 2\pi k}{4} + i \sin \frac{\frac{3\pi}{2} + 2\pi k}{4}\right], k = 0, 1, 2, 3$

$k = 0$: $2\left(\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}\right) \approx 0.7654 + 1.8478i$

$k = 1$: $2\left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}\right) \approx -1.8478 + 0.7654i$

$k = 2$: $2\left(\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}\right) \approx -0.7654 - 1.8478i$

$k = 3$: $2\left(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}\right) \approx 1.8478 - 0.7654i$



110. $x^6 + 64i = 0$

$x^6 = -64i$

The solutions are the sixth roots of $-64i = 64\left[\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right]$:

$\sqrt[6]{64}\left[\cos\left(\frac{(3\pi/2) + 2\pi k}{6}\right) + i \sin\left(\frac{(3\pi/2) + 2\pi k}{6}\right)\right]$

$k = 0, 1, 2, 3, 4, 5$

$k = 0$: $2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \sqrt{2} + \sqrt{2}\, i$

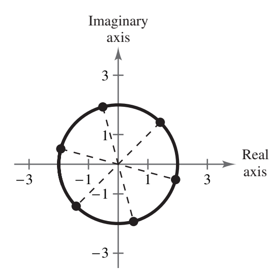
$k = 1$: $2\left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}\right) \approx -0.5176 + 1.9319i$

$k = 2$: $2\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right) \approx -1.9319 + 0.5176i$

$k = 3$: $2\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -\sqrt{2} - \sqrt{2}\, i$

$k = 4$: $2\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right) = 0.5176 - 1.9319i$

$k = 5$: $2\left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12}\right) = 1.9319 - 0.5176i$



111. $x^3 - (1 - i) = 0$

$$x^3 = 1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

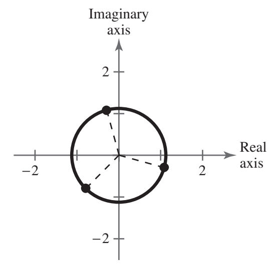
The solutions are the cube roots of $1 - i$:

$$\sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{(7\pi/4) + 2\pi k}{3}\right) + i\sin\left(\frac{(7\pi/4) + 2\pi k}{3}\right)\right],\ k = 0, 1, 2$$

$k = 0$: $\sqrt[6]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right) \approx -0.2905 + 1.0842i$

$k = 1$: $\sqrt[6]{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) \approx -0.7937 - 0.7937i$

$k = 2$: $\sqrt[6]{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) \approx 1.0842 - 0.2905i$



112. $x^4 + (1 + i) = 0$

$$x^4 = -1 - i = \sqrt{2}(\cos 225° + i\sin 225°)$$

The solutions are the fourth roots of $-1 - i$:

$$\sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°k}{4}\right) + i\sin\left(\frac{225° + 360°k}{4}\right)\right]$$

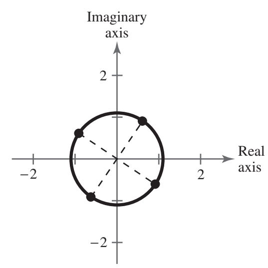
$k = 0, 1, 2, 3$

$k = 0$: $\sqrt[8]{2}(\cos 56.25° + i\sin 56.25°) \approx 0.6059 + 0.9067i$

$k = 1$: $\sqrt[8]{2}(\cos 146.25° + i\sin 146.25°) \approx -0.9067 + 0.6059i$

$k = 2$: $\sqrt[8]{2}(\cos 236.25° + i\sin 236.25°) \approx -0.6059 - 0.9067i$

$k = 3$: $\sqrt[8]{2}(\cos 326.25° + i\sin 326.25°) \approx 0.9067 - 0.6059i$



113. True, by the definition of the absolute value of a complex number.

114. False. They are equally spaced along the circle centered at the origin with radius $\sqrt[n]{r}$.

115. True. $z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$ and $z_1z_2 = 0$ if and only if $r_1 = 0$ and/or $r_2 = 0$.

116. False. The complex number must be converted to trigonometric form before applying DeMoivre's Theorem.

$$\left(4 + \sqrt{6}i\right)^8 = \left[\sqrt{22}\left(\cos\left(\arctan\frac{\sqrt{6}}{4}\right) + i\sin\left(\arctan\frac{\sqrt{6}}{4}\right)\right)\right]^8$$

117. $\dfrac{z_1}{z_2} = \dfrac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} \cdot \dfrac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

$$= \frac{r_1}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1)]$$

$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

118. $\bar{z} = r[\cos(-\theta) + i\sin(-\theta)]$

$= r[\cos\theta + -i\sin\theta]$

$= r\cos\theta - ir\sin\theta$

which is the complex conjugate of $r(\cos\theta + i\sin\theta) = r\cos\theta + ir\sin\theta$.

119. (a) $z\bar{z} = [r(\cos\theta + i\sin\theta)][r(\cos(-\theta) + i\sin(-\theta))]$

$= r^2[\cos(\theta - \theta) + i\sin(\theta - \theta)]$

$= r^2[\cos 0 + i\sin 0]$

$= r^2$

(b) $\dfrac{z}{\bar{z}} = \dfrac{r(\cos\theta + i\sin\theta)}{r[\cos(-\theta) + i\sin(-\theta)]}$

$= \dfrac{r}{r}[\cos(\theta - (-\theta)) + i\sin(\theta - (-\theta))]$

$= \cos 2\theta + i\sin 2\theta$

120. $z = r(\cos\theta + i\sin\theta)$

$-z = -r(\cos\theta + i\sin\theta)$

$= r(-\cos\theta + -i\sin\theta)$

$= r(\cos(\theta + \pi) + i\sin(\theta + \pi))$

121. $-\dfrac{1}{2}(1 + \sqrt{3}i) = -\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

$\left[-\dfrac{1}{2}(1 + \sqrt{3}\mathrm{i})\right]^6 = \left[-\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)\right]^6$

$= \cos 8\pi + i\sin 8\pi$

$= 1$

122. $2^{-1/4}(1 - i) = 2^{-1/4}\left[\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)\right]$

$= 2^{1/4}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$

$\left[2^{1/4}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)\right]^4 = (2^{1/4})^4(\cos 7\pi + i\sin 7\pi)$

$= 2(\cos\pi + i\sin\pi)$

$= -2$

123. (a) $2(\cos 30° + i\sin 30°)$

$2(\cos 150° + i\sin 150°)$

$2(\cos 270° + i\sin 270°)$

(b) These are the cube roots of $8i$.

124. (a) $3(\cos 45° + i\sin 45°)$

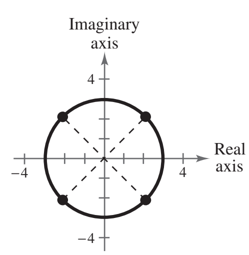
$3(\cos 135° + i\sin 135°)$

$3(\cos 225° + i\sin 225°)$

$3(\cos 315° + i\sin 315°)$

(b) These are the fourth roots of -81.

(c) The fourth roots of -81:



125. $A = 22°, a = 8$

$B = 90° - A = 68°$

$\tan 22° = \dfrac{8}{b} \implies b = \dfrac{8}{\tan 22°} \approx 19.80$

$\sin 22° = \dfrac{8}{c} \implies c = \dfrac{8}{\sin 22°} \approx 21.36$

126. $B = 66°, a = 33.5$

$A = 90° - 66° = 24°$

$b = \dfrac{a \sin B}{\sin A} = \dfrac{(33.5)\sin 66°}{\sin 24°} \approx 75.24$

$c = \dfrac{a \sin C}{\sin A} = \dfrac{(33.5)\sin 90°}{\sin 24°} \approx 82.36$

127. $A = 30°, b = 112.6$

$B = 90° - A = 60°$

$\tan 30° = \dfrac{a}{112.6} \implies a = 112.6 \tan 30° \approx 65.01$

$\cos 30° = \dfrac{112.6}{c} \implies c = \dfrac{112.6}{\cos 30°} \approx 130.02$

128. $B = 6°, b = 211.2$

$A = 90° - 6° = 84°$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{(211.2)\sin 84°}{\sin 6°} \approx 2009.43$

$c = \dfrac{b \sin C}{\sin B} = \dfrac{(211.2)\sin 90°}{\sin 6°} \approx 2020.50$

129. $A = 42°15' = 42.25°, c = 11.2$

$B = 90° - A = 47°45'$

$\sin 42.25° = \dfrac{a}{11.2} \implies a = 11.2 \sin 42.25° \approx 7.53$

$\cos 42.25° = \dfrac{b}{11.2} \implies b = 11.2 \cos 42.25° \approx 8.29$

130. $B = 81°\,30', c = 6.8$

$A = 90° - 81°\,30' = 8°\,30'$

$a = \dfrac{c \sin A}{\sin C} = \dfrac{(6.8)\sin 8°\,30'}{1} \approx 1.01$

$b = \dfrac{c \sin B}{\sin C} = \dfrac{(6.8)\sin 81°\,30'}{1} \approx 6.73$

131. $d = 16 \cos \dfrac{\pi}{4}t$

Maximum displacement: $|16| = 16$

$16 \cos \dfrac{\pi}{4}t = 0 \implies \dfrac{\pi}{4}t = \dfrac{\pi}{2} \implies t = 2$

132. $d = \dfrac{1}{8}\cos 12\pi t$

Maximum displacement: $\dfrac{1}{8}$

$d = 0$ when $12\pi t = \dfrac{\pi}{2}$, or $t = \dfrac{1}{24}$

133. $d = \frac{1}{16}\sin\left(\frac{5}{4}\pi t\right)$

Maximum displacement: $\left|\frac{1}{16}\right| = \frac{1}{16}$

$\frac{1}{16}\sin\left(\frac{5}{4}\pi t\right) = 0$

$\frac{5}{4}\pi t = \pi$

$t = \frac{4}{5}$

134. $d = \frac{1}{12}\sin 60\pi t$

Maximum displacement: $\frac{1}{12}$

$d = 0$ when $60\pi t = \pi$, or $t = \frac{1}{60}$

135. $6 \sin 8\theta \cos 3\theta = (6)\left(\frac{1}{2}\right)[\sin(8\theta + 3\theta) + \sin(8\theta - 3\theta)]$

$= 3(\sin 11\theta + \sin 5\theta)$

136. $2 \cos 5\theta \sin 2\theta = 2 \cdot \frac{1}{2}[\sin(5\theta + 2\theta) - \sin(5\theta - 2\theta)]$

$= \sin 7\theta - \sin 3\theta$

Review Exercises for Chapter 6

1. Given: $A = 35°, B = 71°, a = 8$

$C = 180° - 35° - 71° = 74°$

$$b = \frac{a \sin B}{\sin A} = \frac{8 \sin 71°}{\sin 35°} \approx 13.19$$

$$c = \frac{a \sin C}{\sin A} = \frac{8 \sin 74°}{\sin 35°} \approx 13.41$$

2. Given: $A = 22°, B = 121°, a = 17$

$C = 180 - A - B = 37°$

$$b = \frac{a \sin B}{\sin A} = \frac{17 \sin 121°}{\sin 22°} \approx 38.90$$

$$c = \frac{a \sin C}{\sin A} = \frac{17 \sin 37°}{\sin 22°} \approx 27.31$$

3. Given: $B = 72°, C = 82°, b = 54$

$A = 180° - 72° - 82° = 26°$

$$a = \frac{b \sin A}{\sin B} = \frac{54 \sin 26°}{\sin 72°} \approx 24.89$$

$$c = \frac{b \sin C}{\sin B} = \frac{54 \sin 82°}{\sin 72°} \approx 56.23$$

4. Given: $B = 10°, C = 20°, c = 33$

$A = 180° - B - C = 150°$

$$a = \frac{c \sin A}{\sin C} = \frac{33 \sin 150°}{\sin 20°} \approx 48.24$$

$$b = \frac{c \sin B}{\sin C} = \frac{33 \sin 10°}{\sin 20°} \approx 16.75$$

5. Given: $A = 16°, B = 98°, c = 8.4$

$C = 180° - 16° - 98° = 66°$

$$a = \frac{c \sin A}{\sin C} = \frac{8.4 \sin 16°}{\sin 66°} \approx 2.53$$

$$b = \frac{c \sin B}{\sin C} = \frac{8.4 \sin 98°}{\sin 66°} \approx 9.11$$

6. Given: $A = 95°, B = 45°, c = 104.8$

$C = 180° - A - B = 40°$

$$a = \frac{c \sin A}{\sin C} = \frac{104.8 \sin 95°}{\sin 40°} \approx 162.42$$

$$b = \frac{c \sin B}{\sin C} = \frac{104.8 \sin 45°}{\sin 40°} \approx 115.29$$

7. Given: $A = 24°, C = 48°, b = 27.5$

$B = 180° - 24° - 48° = 108°$

$$a = \frac{b \sin A}{\sin B} = \frac{27.5 \sin 24°}{\sin 108°} \approx 11.76$$

$$c = \frac{b \sin C}{\sin B} = \frac{27.5 \sin 48°}{\sin 108°} \approx 21.49$$

8. Given: $B = 64°, C = 36°, a = 367$

$A = 180° - B - C = 80°$

$$b = \frac{a \sin B}{\sin A} = \frac{367 \sin 64°}{\sin 80°} \approx 334.95$$

$$c = \frac{a \sin C}{\sin A} = \frac{367 \sin 36°}{\sin 80°} \approx 219.04$$

9. Given: $B = 150°, b = 30, c = 10$

$$\sin C = \frac{c \sin B}{b} = \frac{10 \sin 150°}{30} \approx 0.1667 \implies C \approx 9.59°$$

$A \approx 180° - 150° - 9.59° = 20.41°$

$$a = \frac{b \sin A}{\sin B} = \frac{30 \sin 20.41°}{\sin 150°} \approx 20.92$$

10. Given: $B = 150°, a = 10, b = 3$

$$\sin A = \frac{a \sin B}{b} = \frac{10 \sin 150°}{3} \approx 1.67 > 1$$

No solution

11. $A = 75°, a = 51.2, b = 33.7$

$$\sin B = \frac{b \sin A}{a} = \frac{33.7 \sin 75°}{51.2} \approx 0.6358 \implies B \approx 39.48°$$

$C \approx 180° - 75° - 39.48° = 65.52°$

$$c = \frac{a \sin C}{\sin A} = \frac{51.2 \sin 65.52°}{\sin 75°} \approx 48.24$$

12. Given: $B = 25°,\ a = 6.2,\ b = 4$

$$\sin A = \frac{a \sin B}{b} \approx 0.65506 \implies A \approx 40.92° \text{ or } 139.08°$$

Case 1: $A \approx 40.92°$

$C \approx 180° - 25° - 40.92° = 114.08°$

$c \approx 8.64$

Case 2: $A \approx 139.08°$

$C \approx 180° - 25° - 139.08° = 15.92°$

$c \approx 2.60$

13. Area $= \frac{1}{2}bc \sin A = \frac{1}{2}(5)(7)\sin 27° \approx 7.9$

14. $B = 80°, a = 4, c = 8$

Area $= \frac{1}{2}ac \sin B = \frac{1}{2}(4)(8)(0.9848) \approx 15.8$

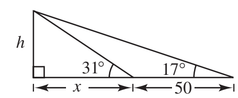
15. Area $= \frac{1}{2}ab \sin C = \frac{1}{2}(16)(5)\sin 123° \approx 33.5$

16. $A = 11°, b = 22, c = 21$

Area $= \frac{1}{2}bc \sin A \approx \frac{1}{2}(22)(21)(0.1908) \approx 44.1$

17. $\tan 17° = \dfrac{h}{x + 50} \implies h = (x + 50) \tan 17°$

$h = x \tan 17° + 50 \tan 17°$



$\tan 31° = \dfrac{h}{x} \implies h = x \tan 31°$

$x \tan 17° + 50 \tan 17° = x \tan 31°$

$50 \tan 17° = x(\tan 31° - \tan 17°)$

$$\frac{50 \tan 17°}{\tan 31° - \tan 17°} = x$$

$x \approx 51.7959$

$h = x \tan 31° \approx 51.7959 \tan 31° \approx 31.1$ meters

The height of the building is approximately 31.1 meters.

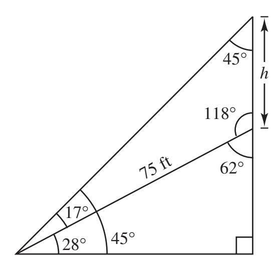
18. $16^2 = w^2 + 12^2 - 2w(12) \cos 140°$

$w^2 - (24 \cos 140°)w - 112 = 0 \implies w \approx 4.83$

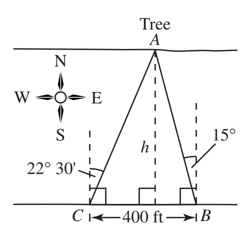
19. $\dfrac{h}{\sin 17°} = \dfrac{75}{\sin 45°}$

$h = \dfrac{75 \sin 17°}{\sin 45°}$

$h \approx 31.01$ feet



20. The triangle of base 400 feet formed by the two angles of sight to the tree has base angles of $90° - 22° 30' = 67° 30'$, or $67.5°$, and $90° - 15° = 75°$. The angle at the tree measures $180° - 67.5° - 75° = 37.5°$.



$$b = \frac{400 \sin 75°}{\sin 37.5°} \approx 634.683$$

$h = 634.683 \sin 67.5°$

$h \approx 586.4$

The width of the river is about 586.4 feet.

21. Given: $a = 5, b = 8, c = 10$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = -0.1375 \implies C \approx 97.90^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = 0.61 \implies B \approx 52.41^\circ$$

$$A = 180^\circ - B - C \approx 29.69^\circ$$

22. Given: $a = 80, b = 60, c = 100$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{6400 + 3600 - 10{,}000}{2(80)(60)}$$

$$= 0 \implies C = 90^\circ$$

$$\sin A = \frac{80}{100} = 0.8 \implies A \approx 53.13^\circ$$

$$\sin B = \frac{60}{100} = 0.6 \implies B \approx 36.87^\circ$$

23. Given: $a = 2.5, b = 5.0, c = 4.5$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = 0.0667 \implies B \approx 86.18^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = 0.44 \implies C \approx 63.90^\circ$$

$$A = 180^\circ - B - C \approx 29.92^\circ$$

24. Given: $a = 16.4, b = 8.8, c = 12.2$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8.8^2 + 12.2^2 - 16.4^2}{2(8.8)(12.2)} \approx -0.1988 \implies A \approx 101.47^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{8.8 \sin 101.47^\circ}{16.4} \approx 0.5259 \implies B \approx 31.73^\circ$$

$$C \approx 180^\circ - 101.47^\circ - 31.73^\circ = 46.80^\circ$$

25. Given: $B = 110^\circ, a = 4, c = 4$

$$b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx 6.55$$

$$A = C = \tfrac{1}{2}(180^\circ - 110^\circ) = 35^\circ$$

26. Given: $B = 150^\circ, a = 10, c = 20$

$$b^2 = 10^2 + 20^2 - 2(10)(20)\cos 150^\circ \implies b \approx 29.09$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{10 \sin 150^\circ}{29.09} \implies A \approx 9.90^\circ$$

$$C \approx 180^\circ - 150^\circ - 9.90^\circ = 20.10^\circ$$

27. Given: $C = 43^\circ, a = 22.5, b = 31.4$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx 21.42$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx -0.02169 \implies B \approx 91.24^\circ$$

$$A = 180^\circ - B - C \approx 45.76^\circ$$

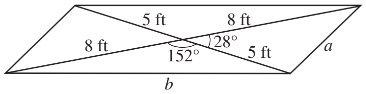
28. Given: $A = 62^\circ, b = 11.34, c = 19.52$

$$a^2 = 11.34^2 + 19.52^2 - 2(11.34)(19.52) \cos 62^\circ \implies a \approx 17.37$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{11.34 \sin 62^\circ}{17.37} \implies B \approx 35.20^\circ$$

$$C \approx 180^\circ - 62^\circ - 35.20^\circ = 82.80^\circ$$

29.



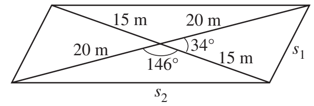
$a^2 = 5^2 + 8^2 - 2(5)(8)\cos 28° \approx 18.364$

$a \approx 4.3$ feet

$b^2 = 8^2 + 5^2 - 2(8)(5)\cos 152° \approx 159.636$

$b \approx 12.6$ feet

30.



${s_1}^2 = 15^2 + 20^2 + 2 \cdot 15 \cdot 20 \cos 34° \approx 127.58$

$s_1 \approx 11.3$ meters

${s_2}^2 = 15^2 + 20^2 + 2 \cdot 15 \cdot 20 \cos 146° \approx 1122.42$

$s_2 \approx 33.5$ meters

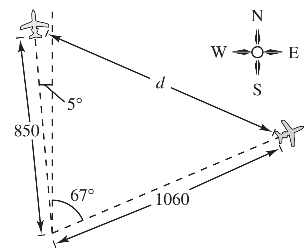
31. Length of AC $= \sqrt{300^2 + 425^2 - 2(300)(425)\cos 115°}$

≈ 615.1 meters

32. $d^2 = 850^2 + 1060^2 - 2(850)(1060)\cos 72°$

$\approx 1{,}289{,}251$

$d \approx 1135$ miles



33. $a = 4,\ b = 5,\ c = 7$

$s = \dfrac{a+b+c}{2} = \dfrac{4+5+7}{2} = 8$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(4)(3)(1)} \approx 9.80$

34. $a = 15, b = 8, c = 10$

$s = \dfrac{15+8+10}{2} = 16.5$

Area $= \sqrt{16.5(1.5)(8.5)(6.5)} \approx 36.979$

35. $a = 12.3, b = 15.8, c = 3.7$

$s = \dfrac{a+b+c}{2} = \dfrac{12.3+15.8+3.7}{2} = 15.9$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15.9(3.6)(0.1)(12.2)} = 8.36$

36. $a = 38.1, b = 26.7, c = 19.4$

$s = \dfrac{38.1+26.7+19.4}{2} = 42.1$

Area $= \sqrt{42.1(4)(15.4)(22.7)} \approx 242.630$

37. $\|\mathbf{u}\| = \sqrt{(4-(-2))^2 + (6-1)^2} = \sqrt{61}$

$\|\mathbf{v}\| = \sqrt{(6-0)^2 + (3-(-2))^2} = \sqrt{61}$

$\mathbf{u}$ is directed along a line with a slope of $\dfrac{6-1}{4-(-2)} = \dfrac{5}{6}$.

$\mathbf{v}$ is directed along a line with a slope of $\dfrac{3-(-2)}{6-0} = \dfrac{5}{6}$.

Since $\mathbf{u}$ and $\mathbf{v}$ have identical magnitudes and directions, $\mathbf{u} = \mathbf{v}$.

38. $\|\mathbf{u}\| = \sqrt{(3-1)^2 + (-2-4)^2} = 2\sqrt{10}$

$\|\mathbf{v}\| = \sqrt{(-1-(-3))^2 + (-4-2)^2} = 2\sqrt{10}$

$\mathbf{u}$ is directed along a line with a slope of $\dfrac{-2-4}{3-1} = -3$.

$\mathbf{v}$ is directed along a line with a slope of $\dfrac{-4-2}{-1-(-3)} = -3$.

Since $\mathbf{u}$ and $\mathbf{v}$ have identical magnitudes and directions, $\mathbf{u} = \mathbf{v}$.

39. Initial point: $(-5, 4)$

Terminal point: $(2, -1)$

$\mathbf{v} = \langle 2-(-5), -1-4 \rangle = \langle 7, -5 \rangle$

40. Initial point: $(0, 1)$

Terminal point: $\left(6, \frac{7}{2}\right)$

$\mathbf{v} = \left\langle 6-0, \frac{7}{2}-1 \right\rangle = \left\langle 6, \frac{5}{2} \right\rangle$

41. Initial point: $(0, 10)$

Terminal point: $(7, 3)$

$\mathbf{v} = \langle 7 - 0, 3 - 10 \rangle = \langle 7, -7 \rangle$

42. Initial point: $(1, 5)$

Terminal point: $(15, 9)$

$\mathbf{v} = \langle 15 - 1, 9 - 5 \rangle = \langle 14, 4 \rangle$

43. $\|\mathbf{v}\| = 8, \theta = 120°$

$\langle 8 \cos 120°, 8 \sin 120° \rangle = \left\langle -4, 4\sqrt{3} \right\rangle$

44. $\|\mathbf{v}\| = \frac{1}{2}, \theta = 225°$

$\left\langle \frac{1}{2} \cos 225°, \frac{1}{2} \sin 225° \right\rangle = \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4} \right\rangle$

45. $\mathbf{u} = \langle -1, -3 \rangle, \mathbf{v} = \langle -3, 6 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -1, -3 \rangle + \langle -3, 6 \rangle = \langle -4, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -1, -3 \rangle - \langle -3, 6 \rangle = \langle 2, -9 \rangle$

(c) $3\mathbf{u} = 3\langle -1, -3 \rangle = \langle -3, -9 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = 2\langle -3, 6 \rangle + 5\langle -1, -3 \rangle$

$= \langle -6, 12 \rangle + \langle -5, -15 \rangle = \langle -11, -3 \rangle$

46. $\mathbf{u} = \langle 4, 5 \rangle, \mathbf{v} = \langle 0, -1 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 4 + 0, 5 + (-1) \rangle = \langle 4, 4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 4 - 0, 5 - (-1) \rangle = \langle 4, 6 \rangle$

(c) $3\mathbf{u} = \langle 3(4), 3(5) \rangle = \langle 12, 15 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 2(0), 2(-1) \rangle + \langle 5(4), 5(5) \rangle$

$= \langle 0 + 20, -2 + 25 \rangle = \langle 20, 23 \rangle$

47. $\mathbf{u} = \langle -5, 2 \rangle, \mathbf{v} = \langle 4, 4 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -5, 2 \rangle + \langle 4, 4 \rangle = \langle -1, 6 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -5, 2 \rangle - \langle 4, 4 \rangle = \langle -9, -2 \rangle$

(c) $3\mathbf{u} = 3\langle -5, 2 \rangle = \langle -15, 6 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = 2\langle 4, 4 \rangle + 5\langle -5, 2 \rangle$

$= \langle 8, 8 \rangle + \langle -25, 10 \rangle = \langle -17, 18 \rangle$

48. $\mathbf{u} = \langle 1, -8 \rangle, \mathbf{v} = \langle 3, -2 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 1 + 3, -8 + (-2) \rangle = \langle 4, -10 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 1 - 3, -8 - (-2) \rangle = \langle -2, -6 \rangle$

(c) $3\mathbf{u} = \langle 3(1), 3(-8) \rangle = \langle 3, -24 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 2(3), 2(-2) \rangle + \langle 5(1), 5(-8) \rangle$

$= \langle 6 + 5, -4 + (-40) \rangle = \langle 11, -44 \rangle$

49. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (5\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} + 2\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = (2\mathbf{i} - \mathbf{j}) - (5\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} - 4\mathbf{j}$

(c) $3\mathbf{u} = 3(2\mathbf{i} - \mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$

(d) $2\mathbf{v} + 5\mathbf{u} = 2(5\mathbf{i} + 3\mathbf{j}) + 5(2\mathbf{i} - \mathbf{j})$

$= (10\mathbf{i} + 6\mathbf{j}) + (10\mathbf{i} - 5\mathbf{j}) = 20\mathbf{i} + \mathbf{j}$

50. $\mathbf{u} = -7\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} - \mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = -7\mathbf{i} - 3\mathbf{j} + 4\mathbf{i} - \mathbf{j} = -3\mathbf{i} - 4\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = -7\mathbf{i} - 3\mathbf{j} - 4\mathbf{i} + \mathbf{j} = -11\mathbf{i} - 2\mathbf{j}$

(c) $3\mathbf{u} = 3(-7\mathbf{i} - 3\mathbf{j}) = -21\mathbf{i} - 9\mathbf{j}$

(d) $2\mathbf{v} + 5\mathbf{u} = 8\mathbf{i} - 2\mathbf{j} - 35\mathbf{i} - 15\mathbf{j}$

$= -27\mathbf{i} - 17\mathbf{j}$

51. $\mathbf{u} = 4\mathbf{i}, \mathbf{v} = -\mathbf{i} + 6\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 4\mathbf{i} + (-\mathbf{i} + 6\mathbf{j}) = 3\mathbf{i} + 6\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = 4\mathbf{i} - (-\mathbf{i} + 6\mathbf{j}) = 5\mathbf{i} - 6\mathbf{j}$

(c) $3\mathbf{u} = 3(4\mathbf{i}) = 12\mathbf{i}$

(d) $2\mathbf{v} + 5\mathbf{u} = 2(-\mathbf{i} + 6\mathbf{j}) + 5(4\mathbf{i})$

$= (-2\mathbf{i} + 12\mathbf{j}) + 20\mathbf{i} = 18\mathbf{i} + 12\mathbf{j}$

52. $\mathbf{u} = -6\mathbf{j}, \mathbf{v} = \mathbf{i} + \mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = -6\mathbf{j} + \mathbf{i} + \mathbf{j} = \mathbf{i} - 5\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = -6\mathbf{j} - \mathbf{i} - \mathbf{j} = -\mathbf{i} - 7\mathbf{j}$

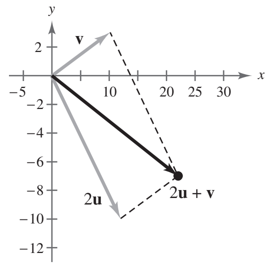
(c) $3\mathbf{u} = -18\mathbf{j}$

(d) $2\mathbf{v} + 5\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} - 30\mathbf{j}$

$= 2\mathbf{i} - 28\mathbf{j}$

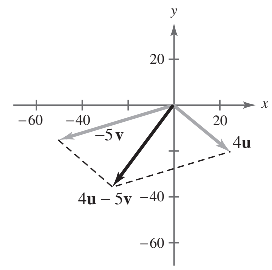
53. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$$2\mathbf{u} + \mathbf{v} = 2(6\mathbf{i} - 5\mathbf{j}) + (10\mathbf{i} + 3\mathbf{j})$$
$$= 22\mathbf{i} - 7\mathbf{j}$$
$$= \langle 22, -7\rangle$$



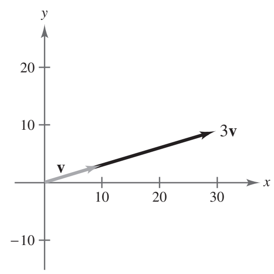
54. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \ \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$$4\mathbf{u} - 5\mathbf{v} = (24\mathbf{i} - 20\mathbf{j}) - (50\mathbf{i} + 15\mathbf{j})$$
$$= -26\mathbf{i} - 35\mathbf{j}$$
$$= \langle -26, -35\rangle$$



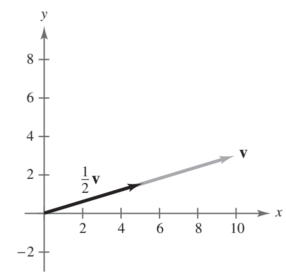
55. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$$3\mathbf{v} = 3(10\mathbf{i} + 3\mathbf{j})$$
$$= 30\mathbf{i} + 9\mathbf{j}$$
$$= \langle 30, 9\rangle$$



56. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$$\tfrac{1}{2}\mathbf{v} = 5\mathbf{i} + \tfrac{3}{2}\mathbf{j} = \left\langle 5, \tfrac{3}{2}\right\rangle$$



57. $\mathbf{u} = \langle -3, 4\rangle = -3\mathbf{i} + 4\mathbf{j}$

58. $\mathbf{u} = \langle -6, -8\rangle = -6\mathbf{i} - 8\mathbf{j}$

59. Initial point: $(3, 4)$

Terminal point: $(9, 8)$

$\mathbf{u} = (9 - 3)\mathbf{i} + (8 - 4)\mathbf{j} = 6\mathbf{i} + 4\mathbf{j}$

60. Initial point: $(-2, 7)$

Terminal point: $(5, -9)$

$\mathbf{u} = \langle 5 - (-2), -9 - 7\rangle = \langle 7, -16\rangle = 7\mathbf{i} - 16\mathbf{j}$

61. $\mathbf{v} = -10\mathbf{i} + 10\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-10)^2 + (10)^2} = \sqrt{200} = 10\sqrt{2}$

$\tan\theta = \dfrac{10}{-10} = -1 \implies \theta = 135°$ since

v is in Quadrant II.

$\mathbf{v} = 10\sqrt{2}(\mathbf{i}\cos 135° + \mathbf{j}\sin 135°)$

62. $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

$\|\mathbf{v}\| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$

$\tan\theta = \dfrac{-1}{4}$, θ in Quadrant IV $\implies \theta \approx 346°$

$\mathbf{v} \approx \sqrt{17}(\cos 346°\,\mathbf{i} + \sin 346°\,\mathbf{j})$

63. $\mathbf{v} = 7(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j})$

$\|\mathbf{v}\| = 7$

$\theta = 60°$

64. $\mathbf{v} = 3(\cos 150°\mathbf{i} + \sin 150°\,\mathbf{j})$

$\|\mathbf{v}\| = 3, \theta = 150°$

65. $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{5^2 + 4^2} = \sqrt{41}$

$\tan\theta = \dfrac{4}{5} \implies \theta \approx 38.7°$

66. $\mathbf{v} = -4\mathbf{i} + 7\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-4)^2 + 7^2} = \sqrt{65}$

$\tan\theta = \dfrac{7}{-4}$, θ in Quadrant II $\implies \theta \approx 119.7°$

67. $\mathbf{v} = -3\mathbf{i} - 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$

$\tan\theta = \dfrac{-3}{-3} = 1 \implies \theta = 225°$

68. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$

$\|\mathbf{v}\| = \sqrt{8^2 + (-1)^2} = \sqrt{65}$

$\tan\theta = \dfrac{-1}{8}$, θ in Quadrant IV $\implies \theta \approx 352.9°$

69. Magnitude of resultant:

$$c = \sqrt{85^2 + 50^2 - 2(85)(50)\cos 165^\circ}$$
$$\approx 133.92 \text{ pounds}$$

Let θ be the angle between the resultant and the 85-pound force.

$$\cos\theta \approx \frac{(133.92)^2 + 85^2 - 50^2}{2(133.92)(85)}$$
$$\approx 0.9953$$
$$\Longrightarrow \theta \approx 5.6^\circ$$

70. Rope One:

$$\mathbf{u} = \|\mathbf{u}\|(\cos 30^\circ\mathbf{i} - \sin 30^\circ\mathbf{j}) = \|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

Rope Two:

$$\mathbf{v} = \|\mathbf{u}\|(-\cos 30^\circ\mathbf{i} - \sin 30^\circ\mathbf{j}) = \|\mathbf{u}\|\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

Resultant: $\mathbf{u} + \mathbf{v} = -\|\mathbf{u}\|\mathbf{j} = -180\mathbf{j}$

$$\|\mathbf{u}\| = 180$$

Therefore, the tension on each rope is $\|\mathbf{u}\| = 180$ lb.

71. Airspeed: $\mathbf{u} = 430(\cos 45^\circ\mathbf{i} - \sin 45^\circ\mathbf{j}) = 215\sqrt{2}(\mathbf{i} - \mathbf{j})$

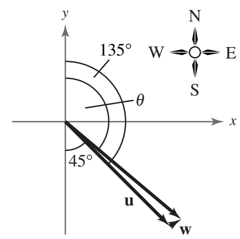
Wind: $\mathbf{w} = 35(\cos 60^\circ + \sin 60^\circ\,\mathbf{j}) = \dfrac{35}{2}(\mathbf{i} + \sqrt{3}\mathbf{j})$

Groundspeed: $\mathbf{u} + \mathbf{w} = \left(215\sqrt{2} + \dfrac{35}{2}\right)\mathbf{i} + \left(\dfrac{35\sqrt{3}}{2} - 215\sqrt{2}\right)$

$$\|\mathbf{u} + \mathbf{w}\| = \sqrt{\left(215\sqrt{2} + \frac{35}{2}\right)^2 + \left(\frac{35\sqrt{3}}{2} - 215\sqrt{2}\right)^2}$$
$$\approx 422.30 \text{ miles per hour}$$

Bearing: $\tan\theta' = \dfrac{17.5\sqrt{3} - 215\sqrt{2}}{215\sqrt{2} + 17.5}$

$$\theta' \approx -40.4^\circ$$
$$\theta = 90^\circ + |\theta'| = 130.4^\circ$$



72. Airspeed: $\mathbf{u} = 724(\cos 60^\circ\mathbf{i} + \sin 60^\circ\mathbf{j})$

$$= 362(\mathbf{i} + \sqrt{3}\mathbf{j})$$

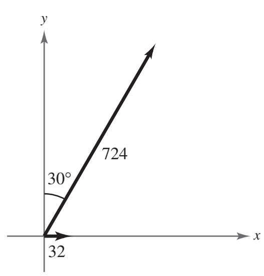
Wind: $\mathbf{w} = 32\mathbf{i}$

Groundspeed $= \mathbf{u} + \mathbf{w} = (394\mathbf{i} + 362\sqrt{3}\mathbf{j})$

$\|\mathbf{u} + \mathbf{w}\| = \sqrt{(394)^2 + (362\sqrt{3})^2} \approx 740.5$ km/hr

$\tan\theta = \dfrac{362\sqrt{3}}{394} \Longrightarrow \theta \approx 57.9^\circ$

Bearing: N 32.1° E



73. $\mathbf{u} = \langle 6, 7\rangle$, $\mathbf{v} = \langle -3, 9\rangle$

$\mathbf{u}\cdot\mathbf{v} = 6(-3) + 7(9) = 45$

74. $\mathbf{u} = \langle -7, 12\rangle$, $\mathbf{v} = \langle -4, -14\rangle$

$\mathbf{u}\cdot\mathbf{v} = -7(-4) + 12(-14) = -140$

75. $\mathbf{u} = 3\mathbf{i} + 7\mathbf{j}$, $\mathbf{v} = 11\mathbf{i} - 5\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = 3(11) + 7(-5) = -2$

76. $\mathbf{u} = -7\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 16\mathbf{i} - 12\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = -7(16) + 2(-12) = -136$

77. $\mathbf{u} = \langle -3, 4\rangle$

$2\mathbf{u} = \langle -6, 8\rangle$

$2\mathbf{u}\cdot\mathbf{u} = (-6)(-3) + 8(4) = 50$

The result is a scalar.

78. $\mathbf{v} = \langle 2, 1\rangle$

$\|\mathbf{v}\|^2 = \mathbf{v}\cdot\mathbf{v} = 2^2 + 1^2 = 5$; scalar

79. $\mathbf{u} = \langle -3, 4\rangle, \mathbf{v} = \langle 2, 1\rangle$

$\mathbf{u}\cdot\mathbf{v} = (-3)(2) + 4(1) = -2$

$\mathbf{u}(\mathbf{u}\cdot\mathbf{v}) = \mathbf{u}(-2) = -2\mathbf{u} = \langle 6, -8\rangle$

The result is a vector.

80. $\mathbf{u} = \langle -3, 4\rangle, \mathbf{v} = \langle 2, 1\rangle$

$3\mathbf{u}\cdot\mathbf{v} = 3(-3(2) + 4(1)) = 3(-2) = -6$; scalar

81. $\mathbf{u} = \cos\frac{7\pi}{4}\mathbf{i} + \sin\frac{7\pi}{4}\mathbf{j} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\rangle$

$\mathbf{v} = \cos\frac{5\pi}{6}\mathbf{i} + \sin\frac{5\pi}{6}\mathbf{j} = \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-\sqrt{3}-1}{2\sqrt{2}} \implies \theta = \frac{11\pi}{12}$

82. $\mathbf{u} = \cos 45^\circ\,\mathbf{i} + \sin 45^\circ\,\mathbf{j}$

$\mathbf{v} = \cos 300^\circ\,\mathbf{i} + \sin 300^\circ\,\mathbf{j}$

Angle between **u** and **v**: $60^\circ + 45^\circ = 105^\circ$

83. $\mathbf{u} = \left\langle 2\sqrt{2}, -4\right\rangle, \mathbf{v} = \left\langle -\sqrt{2}, 1\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-8}{\left(\sqrt{24}\right)\left(\sqrt{3}\right)} \implies \theta \approx 160.5^\circ$

84. $\mathbf{u} = \left\langle 3, \sqrt{3}\right\rangle, \mathbf{v} = \left\langle 4, 3\sqrt{3}\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{21}{\sqrt{12}\sqrt{43}} \implies \theta \approx 22.4^\circ$

85. $\mathbf{u} = \langle -3, 8\rangle$

$\mathbf{v} = \langle 8, 3\rangle$

$\mathbf{u}\cdot\mathbf{v} = -3(8) + 8(3) = 0$

u and **v** are orthogonal.

86. $\mathbf{u} = \left\langle \frac{1}{4}, -\frac{1}{2}\right\rangle, \mathbf{v} = \langle -2, 4\rangle$

$\mathbf{v} = -8\mathbf{u} \implies$ Parallel

87. $\mathbf{u} = -\mathbf{i}$

$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} \neq 0 \implies$ Not orthogonal

$\mathbf{v} \neq k\mathbf{u} \implies$ Not parallel

Neither

88. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}, \mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ Orthogonal

89. $\mathbf{u} = \langle -4, 3\rangle,\ \mathbf{v} = \langle -8, -2\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{26}{68}\right)\langle -8, -2\rangle = -\frac{13}{17}\langle 4, 1\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -4, 3\rangle - \left(-\frac{13}{17}\right)\langle 4, 1\rangle = \frac{16}{17}\langle -1, 4\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = -\frac{13}{17}\langle 4, 1\rangle + \frac{16}{17}\langle -1, 4\rangle$

90. $\mathbf{u} = \langle 5, 6\rangle, \mathbf{v} = \langle 10, 0\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{50}{100}\langle 10, 0\rangle = \langle 5, 0\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 5, 6\rangle - \langle 5, 0\rangle = \langle 0, 6\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \langle 5, 0\rangle + \langle 0, 6\rangle$

91. $\mathbf{u} = \langle 2, 7\rangle,\ \mathbf{v} = \langle 1, -1\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = -\frac{5}{2}\langle 1, -1\rangle$

$= \frac{5}{2}\langle -1, 1\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 7\rangle - \left(\frac{5}{2}\right)\langle -1, 1\rangle$

$= \frac{9}{2}\langle 1, 1\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{5}{2}\langle -1, 1\rangle + \frac{9}{2}\langle 1, 1\rangle$

92. $\mathbf{u} = \langle -3, 5\rangle, \mathbf{v} = \langle -5, 2\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{25}{29}\langle -5, 2\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -3, 5\rangle - \frac{25}{29}\langle -5, 2\rangle = \frac{19}{29}\langle 2, 5\rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{25}{29}\langle -5, 2\rangle + \frac{19}{25}\langle 2, 5\rangle$$

93. $P = (5, 3), Q = (8, 9) \implies \overrightarrow{PQ} = \langle 3, 6\rangle$

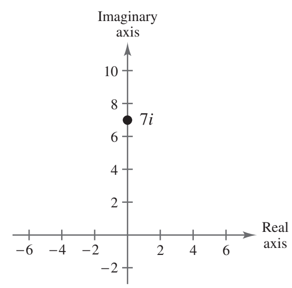
$$W = \mathbf{v}\cdot\overrightarrow{PQ} = \langle 2, 7\rangle \cdot \langle 3, 6\rangle = 48$$

94.
$$\begin{aligned}\text{work} &= \mathbf{v}\cdot\overrightarrow{PQ}\\ &= (3\mathbf{i} - 6\mathbf{j})\cdot(-10\mathbf{i} + 17\mathbf{j})\\ &= -30 - 102\\ &= -132\end{aligned}$$

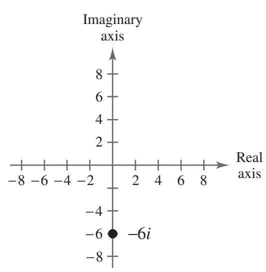
95. $w = (18{,}000)\left(\frac{48}{12}\right) = 72{,}000$ foot-pounds

96.
$$\begin{aligned}W &= \cos\theta\,\|\mathbf{F}\|\,\|\overrightarrow{PQ}\|\\ &= (\cos 20°)(25 \text{ pounds})(12 \text{ ft})\\ &= 281.9 \text{ foot-pounds}\end{aligned}$$

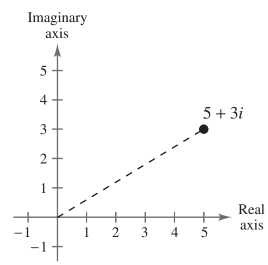
97. $|7i| = \sqrt{0^2 + 7^2} = 7$



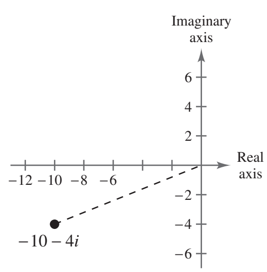
98. $|-6i| = 6$



99. $|5 + 3i| = \sqrt{5^2 + 3^2}$
$= \sqrt{34}$



100. $|-10 - 4i| = \sqrt{(-10)^2 + (-4)^2}$
$= 2\sqrt{29}$



101. $5 - 5i$

$$r = \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

$\tan\theta = \dfrac{-5}{5} = -1 \implies \theta = \dfrac{7\pi}{4}$ since the complex number is in Quadrant IV.

$$5 - 5i = 5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

102. $z = 5 + 12i$

$$|z| = \sqrt{5^2 + 12^2} = 13$$

$$\tan\theta = \frac{12}{5} \implies \theta \approx 1.176$$

$$z \approx 13(\cos 1.176 + i\sin 1.176)$$

103. $-3\sqrt{3} + 3i$

$$r = \sqrt{\left(-3\sqrt{3}\right)^2 + 3^2} = \sqrt{36} = 6$$

$\tan\theta = \dfrac{3}{-3\sqrt{3}} = -\dfrac{1}{\sqrt{3}} \implies \theta = \dfrac{5\pi}{6}$

since the complex number is in Quadrant II.

$$-3\sqrt{3} + 3i = 6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

104. $z = -7$

$|z| = 7$

$\tan\theta = \dfrac{0}{-7} = 0 \implies \theta = \pi$

$z = 7(\cos\pi + i\sin\pi)$

105. (a) $z_1 = 2\sqrt{3} - 2i = 4\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)$

$z_2 = -10i = 10\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$

(b) $z_1z_2 = \left[4\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)\right]\left[10\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)\right]$

$= 40\left(\cos\dfrac{10\pi}{3} + i\sin\dfrac{10\pi}{3}\right)$

$\dfrac{z_1}{z_2} = \dfrac{4\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)}{10\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)} = \dfrac{2}{5}\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$

106. (a) $z_1 = -3(1 + i) = 3\sqrt{2}\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$

$z_2 = 2\left(\sqrt{3} + i\right) = 4\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

(b) $z_1z_2 = \left[3\sqrt{2}\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)\right]\left[4\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)\right]$

$= 12\sqrt{2}\left(\cos\dfrac{17\pi}{12} + i\sin\dfrac{17\pi}{12}\right)$

$\dfrac{z_1}{z_2} = \dfrac{3\sqrt{2}\left[\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right]}{4\left[\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right]} = \dfrac{3\sqrt{2}}{4}\left(\cos\dfrac{13\pi}{12} + i\sin\dfrac{13\pi}{12}\right)$

107. $\left[5\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)\right]^4 = 5^4\left(\cos\dfrac{4\pi}{12} + i\sin\dfrac{4\pi}{12}\right)$

$= 625\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$

$= 625\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)$

$= \dfrac{625}{2} + \dfrac{625\sqrt{3}}{2}i$

108. $\left[2\left(\cos\dfrac{4\pi}{15} + i\sin\dfrac{4\pi}{15}\right)\right]^5 = 2^5\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

$= 32\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right)$

$= -16 - 16\sqrt{3}i$

109. $(2 + 3i)^6 \approx [\sqrt{13}(\cos 56.3° + i\sin 56.3°)]^6$

$= 13^3(\cos 337.9° + i\sin 337.9°)$

$\approx 13^3(0.9263 - 0.3769i)$

$\approx 2035 - 828i$

110. $(1 - i)^8 = \left[\sqrt{2}(\cos 315° + i\sin 315°)\right]^8$

$= 16(\cos 2520° + i\sin 2520°)$

$= 16(\cos 0° + i\sin 0°)$

$= 16$

111. Sixth roots of $-729i = 729\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$:

(a) and (c)

$$\sqrt[6]{729}\left[\cos\left(\frac{\frac{3\pi}{2} + 2k\pi}{6}\right) + i\sin\left(\frac{\frac{3\pi}{2} + 2k\pi}{6}\right)\right], k = 0, 1, 2, 3, 4, 5$$

$k = 0$: $3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

$k = 1$: $3\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right) \approx -0.776 + 2.898i$

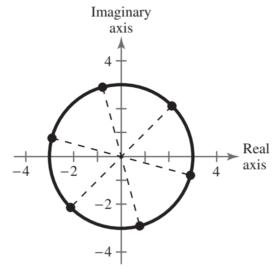
$k = 2$: $3\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) \approx -2.898 + 0.776i$

$k = 3$: $3\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

$k = 4$: $3\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right) \approx 0.776 - 2.898i$

$k = 5$: $3\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) \approx 2.898 - 0.776i$

(b)



112. (a) $256i = 256\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Fourth roots of $256i$:

$$\sqrt[4]{256}\left(\cos\frac{\frac{\pi}{2} + 2\pi k}{4} + i\sin\frac{\frac{\pi}{2} + 2\pi k}{4}\right), k = 0, 1, 2, 3$$

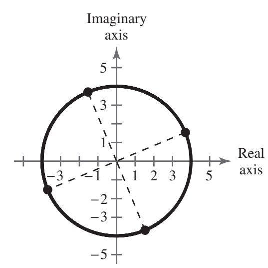
$k = 0$: $4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$k = 1$: $4\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$

$k = 2$: $4\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$

$k = 3$: $4\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$

(b)



(c) $3.696 + 1.531i$

$-1.531 + 3.696i$

$-3.696 - 1.531i$

$1.531 - 3.696i$

113. Cube roots of $8 = 8(\cos 0 + i\sin 0)$, $k = 0, 1, 2$

(a) and (c)

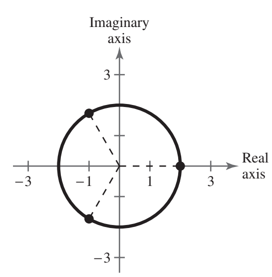
$$\sqrt[3]{8}\left[\cos\left(\frac{0 + 2\pi k}{3}\right) + i\sin\left(\frac{0 + 2\pi k}{3}\right)\right]$$

$k = 0$: $2(\cos 0 + i\sin 0) = 2$

$k = 1$: $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -1 + \sqrt{3}i$

$k = 2$: $2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -1 - \sqrt{3}i$

(b)



114. (a) $-1024 = 1024(\cos \pi + i \sin \pi)$

Fifth roots of -1024:

$$\sqrt[5]{1024}\left(\cos \frac{\pi + 2\pi k}{5} + i \sin\left(\frac{\pi + 2\pi k}{5}\right)\right), k = 0, 1, 2, 3, 4$$

$$k = 0: 4\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

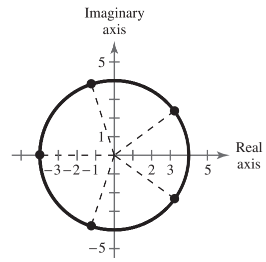
$$k = 1: 4\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$k = 2: 4(\cos \pi + i \sin \pi)$$

$$k = 3: 4\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$k = 4: 4\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$

(b)



(c) $3.236 + 2.351i$

$-1.236 + 3.804i$

-4

$-1.236 - 3.804i$

$3.236 - 2.351i$

115. $x^4 + 81 = 0$

$x^4 = -81$ Solve by finding the fourth roots of -81.

$-81 = 81(\cos \pi + i \sin \pi)$

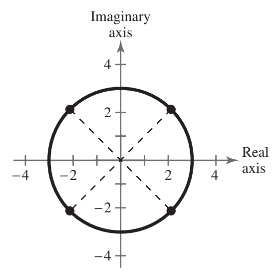
$$\sqrt[4]{-81} = \sqrt[4]{81}\left[\cos\left(\frac{\pi + 2\pi k}{4}\right) + i \sin\left(\frac{\pi + 2\pi k}{4}\right)\right],\ k = 0, 1, 2, 3$$

$$k = 0: 3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

$$k = 1: 3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

$$k = 2: 3\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

$$k = 3: 3\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$



116. $x^5 - 32 = 0$

$x^5 = 32$

$32 = 32(\cos 0 + i \sin 0)$

$$\sqrt[3]{32} = \sqrt[5]{32}\left[\cos\left(0 + \frac{2\pi k}{5}\right) + i \sin\left(0 + \frac{2\pi k}{5}\right)\right]$$

$k = 0, 1, 2, 3, 4$

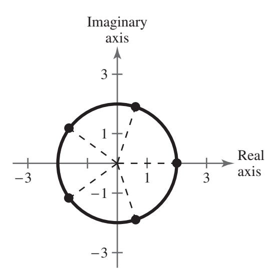
$k = 0$: $2(\cos 0 + i \sin 0) = 2$

$$k = 1:\ 2\left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right) = 0.6180 + 1.9021i$$

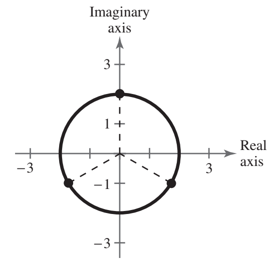
$$k = 2:\ 2\left(\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}\right) = -1.6180 + 1.1756i$$

$$k = 3:\ 2\left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}\right) = -1.6180 - 1.1756i$$

$$k = 4:\ 2\left(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}\right) = 0.6180 - 1.9021i$$



117. $x^3 + 8i = 0$



$x^3 = -8i$ Solve by finding the cube roots of $-8i$.

$$-8i = 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

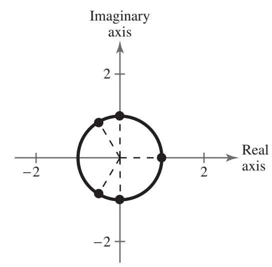
$$\sqrt[3]{-8i} = \sqrt[3]{8}\left[\cos\left(\frac{\frac{3\pi}{2} + 2\pi k}{3}\right) + i\sin\left(\frac{\frac{3\pi}{2} + 2\pi k}{3}\right)\right],\ k = 0, 1, 2$$

$$k = 0\colon 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$$

$$k = 1\colon 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = -\sqrt{3} - i$$

$$k = 2\colon 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = \sqrt{3} - i$$

118. $(x^3 - 1)(x^2 + 1) = 0$



$$x^3 - 1 = 0$$

$$x^2 + 1 = 0$$

$$x^3 = 1$$

$$1 = 1(\cos 0 + i\sin 0)$$

$$\sqrt[3]{1} = \sqrt[3]{1}\left[\cos\left(\frac{0 + 2\pi k}{3}\right) + i\sin\left(\frac{0 + 2\pi k}{3}\right)\right],\ k = 0, 1, 2$$

$$1(\cos 0 + i\sin 0) = 1$$

$$1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$1\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$-1 = 1(\cos\pi + i\sin\pi)$$

$$\sqrt{-1} = \sqrt{1}\left[\cos\left(\frac{\pi + 2\pi k}{2}\right) + i\sin\left(\frac{\pi + 2\pi k}{2}\right)\right],\ k = 0, 1$$

$$k = 0, 1$$

$$1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = i$$

$$1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -i$$

119. True. $\sin 90°$ is defined in the Law of Sines.

120. False. There may be no solution, one solution, or two solutions.

121. True, by the definition of a unit vector.

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \text{ so } \mathbf{v} = \|\mathbf{v}\|\mathbf{u}$$

122. False, $a = b = 0$.

123. False. $x = \sqrt{3} + i$ is a solution to $x^3 - 8i = 0$, not $x^2 - 8i = 0$.

Also, $(\sqrt{3} + i)^2 - 8i = 2 + (2\sqrt{3} - 8)i \neq 0$.

124. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

125. $a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

126. A vector in the plane has both a magnitude and a direction.

127. A and C appear to have the same magnitude and direction.

128. $\|\mathbf{u} + \mathbf{v}\|$ is larger in figure (a) since the angle between $\mathbf{u}$ and $\mathbf{v}$ is acute rather than obtuse.

129. If $k > 0$, the direction of $k\mathbf{u}$ is the same, and the magnitude is $k\|\mathbf{u}\|$.

If $k < 0$, the direction of $k\mathbf{u}$ is the opposite direction of $\mathbf{u}$, and the magnitude is $|k|\,\|\mathbf{u}\|$.

130. The sum of $\mathbf{u}$ and $\mathbf{v}$ lies on the diagonal of the parallelogram with $\mathbf{u}$ and $\mathbf{v}$ as its adjacent sides.

131. (a) The trigonometric form of the three roots shown is:

$4(\cos 60° + i \sin 60°)$

$4(\cos 180° + i \sin 180°)$

$4(\cos 300° + i \sin 300°)$

(b) Since there are three evenly spaced roots on the circle of radius 4, they are cube roots of a complex number of modulus $4^3 = 64$.

Cubing them yields -64.

$[4(\cos 60° + i \sin 60°)]^3 = -64$

$[4(\cos 180° + i \sin 180°)]^3 = -64$

$[4(\cos 300° + i \sin 300°)]^3 = -64$

132. (a) The trigonometric forms of the four roots shown are:

$4(\cos 60° + i \sin 60°)$

$4(\cos 150° + i \sin 150°)$

$4(\cos 240° + i \sin 240°)$

$4(\cos 330° + i \sin 330°)$

(b) Since there are four evenly spaced roots on the circle of radius 4, they are fourth roots of a complex number of modulus 4^4. In this case, raising them to the fourth power yields $-128 - 128\sqrt{3}i$.

133. $z_1 = 2(\cos\theta + i\sin\theta)$

$z_2 = 2(\cos(\pi - \theta) + i\sin(\pi - \theta))$

$$
\begin{aligned}
z_1 z_2 &= (2)(2)[\cos(\theta + (\pi - \theta)) + i\sin(\theta + (\pi - \theta))] \\
&= 4(\cos\pi + i\sin\pi) \\
&= -4
\end{aligned}
$$

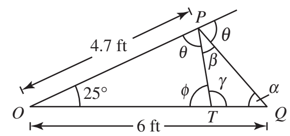
$$
\begin{aligned}
\frac{z_1}{z_2} &= \frac{2(\cos\theta + i\sin\theta)}{2(\cos(\pi - \theta) + i\sin(\pi - \theta))} \\
&= 1[\cos(\theta - (\pi - \theta)) + i\sin(\theta - (\pi - \theta))] \\
&= \cos(2\theta - \pi) + i\sin(2\theta - \pi) \\
&= \cos 2\theta\cos\pi + \sin 2\theta\sin\pi + i(\sin 2\theta\cos\pi - \cos 2\theta\sin\pi) \\
&= -\cos 2\theta - i\sin 2\theta
\end{aligned}
$$

134. (a) z has 4 fourth roots. Three are not shown.

(b) The roots are located on the circle at $\theta = 30° + 90°k$, $k = 0, 1, 2, 3$. The three roots not shown are located at $120°$, $210°$, $300°$.

Problem Solving for Chapter 6

1. $\overrightarrow{PQ}^2 = 4.7^2 + 6^2 - 2(4.7)(6)\cos 25°$



$\overrightarrow{PQ} \approx 2.6409$ feet

$\dfrac{\sin\alpha}{4.7} = \dfrac{\sin 25°}{2.6409} \Rightarrow \alpha \approx 48.78°$

$\theta + \beta = 180° - 25° - 48.78° = 106.22°$

$(\theta + \beta) + \theta = 180° \implies \theta = 180° - 106.22° = 73.78°$

$\beta = 106.22° - 73.78° = 32.44°$

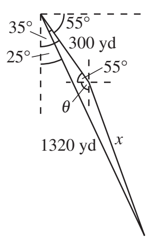
$\gamma = 180° - \alpha - \beta = 180° - 48.78° - 32.44° = 98.78°$

$\phi = 180° - \gamma = 180° - 98.78° = 81.22°$

$\dfrac{\overrightarrow{PT}}{\sin 25°} = \dfrac{4.7}{\sin 81.22°}$

$\overrightarrow{PT} \approx 2.01$ feet

2.



$\frac{3}{4}$ mile $= 1320$ yards

$x^2 = 1320^2 + 300^2 - 2(1320)(300)\cos 10°$

$x \approx 1025.881$ yards ≈ 0.58 mile

$\dfrac{\sin\theta}{1320} = \dfrac{\sin 10°}{1025.881}$

$\sin\theta \approx 0.2234$

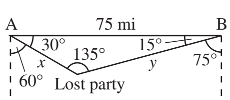
$\theta = 180° - \sin^{-1}(0.2234)$

$\theta \approx 167.09°$

Bearing: $\theta - 55° - 90° \approx 22.09°$

S 22.09° E

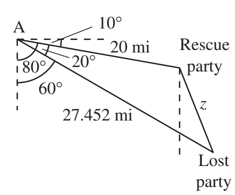
3. (a)



(b) $\dfrac{x}{\sin 15°} = \dfrac{75}{\sin 135°}$ and $\dfrac{y}{\sin 30°} = \dfrac{75}{\sin 135°}$

$x \approx 27.45$ miles $\qquad y \approx 53.03$ miles

(c)



$z^2 = (27.45)^2 + (20)^2 - 2(27.45)(20)\cos 20°$

$z \approx 11.03$ miles

$\dfrac{\sin\theta}{27.45} = \dfrac{\sin 20°}{11.03}$

$\sin\theta \approx 0.8511$

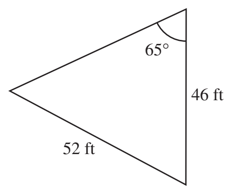
$\theta = 180° - \sin^{-1}(0.8511)$

$\theta \approx 121.7°$

To find the bearing, we have $\theta - 10° - 90° \approx 21.7°$.

Bearing: S 21.7° E

4. (a)



(b) $\dfrac{\sin C}{46} = \dfrac{\sin 65°}{52}$

$\sin C = \dfrac{46 \sin 65°}{52} \approx 0.801734$

$C \approx 53.296°$

$A = 180° - B - C = 61.704°$

$\dfrac{a}{\sin 61.704°} = \dfrac{52}{\sin 65°}$

$a = \dfrac{52 \sin 61.704°}{\sin 65°}$

$a \approx 50.52$ feet

(c) Area $= \frac{1}{2}(46)(52)\sin 61.704° \approx 1053.09$ square feet

Number of bags: $\dfrac{1053.09}{50} \approx 21.06$

To entirely cover the courtyard, you would need to buy 22 bags.

5. If $\mathbf{u} \neq 0$, $\mathbf{v} \neq 0$, and $\mathbf{u} + \mathbf{v} \neq 0$, then $\left\| \dfrac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = \left\| \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = \left\| \dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} \right\| = 1$ since all of these are magnitudes of **unit** vectors.

(a) $\mathbf{u} = \langle 1, -1 \rangle, \quad \mathbf{v} = \langle -1, 2 \rangle, \quad \mathbf{u} + \mathbf{v} = \langle 0, 1 \rangle$

$\|\mathbf{u}\| = \sqrt{2}, \quad \|\mathbf{v}\| = \sqrt{5}, \quad \|\mathbf{u} + \mathbf{v}\| = 1$

(b) $\mathbf{u} = \langle 0, 1 \rangle, \quad \mathbf{v} = \langle 3, -3 \rangle, \quad \mathbf{u} + \mathbf{v} = \langle 3, -2 \rangle$

$\|\mathbf{u}\| = 1, \quad \|\mathbf{v}\| = \sqrt{18} = 3\sqrt{2}, \|\mathbf{u} + \mathbf{v}\| = \sqrt{13}$

(c) $\mathbf{u} = \left\langle 1, \frac{1}{2} \right\rangle, \quad \mathbf{v} = \langle 2, 3 \rangle, \ \mathbf{u} + \mathbf{v} = \left\langle 3, \frac{7}{2} \right\rangle$

$\|\mathbf{u}\| = \dfrac{\sqrt{5}}{2}, \quad \|\mathbf{v}\| = \sqrt{13}, \|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + \dfrac{49}{4}} = \dfrac{\sqrt{85}}{2}$

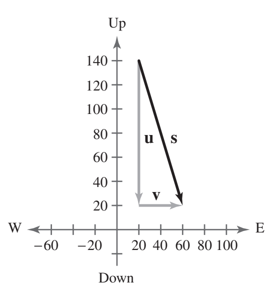
(d) $\mathbf{u} = \langle 2, -4 \rangle, \quad \mathbf{v} = \langle 5, 5 \rangle, \quad \mathbf{u} + \mathbf{v} = \langle 7, 1 \rangle$

$\|\mathbf{u}\| = \sqrt{20} = 2\sqrt{5}, \|\mathbf{v}\| = \sqrt{50} = 5\sqrt{2}, \|\mathbf{u} + \mathbf{v}\| = \sqrt{50} = 5\sqrt{2}$

6. (a) $\mathbf{u} = -120\mathbf{j}$

$\mathbf{v} = 40\mathbf{i}$

(b) $\mathbf{s} = \mathbf{u} + \mathbf{v} = 40\mathbf{i} - 120\mathbf{j}$



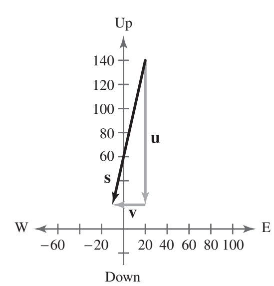
(c) $\|\mathbf{s}\| = \sqrt{40^2 + (-120)^2} = \sqrt{16000} = 40\sqrt{10}$

≈ 126.49 miles per hour

This represents the actual rate of the skydiver's fall.

(d) $\tan\theta = \frac{120}{40} \implies \theta = \tan^{-1} 3 \implies \theta \approx 71.565°$

(e)



$\mathbf{s} = 30\mathbf{i} - 120\mathbf{j}$

$\|\mathbf{s}\| = \sqrt{30^2 + (-120)^2}$

$= \sqrt{15300}$

≈ 123.69 miles per hour

7. Initial point: $(0, 0)$

Terminal point: $\left(\frac{\mathbf{u}_1 + \mathbf{v}_1}{2}, \frac{\mathbf{u}_2 + \mathbf{v}_2}{2}\right)$

$\text{w} = \left\langle \frac{\mathbf{u}_1 + \mathbf{v}_1}{2}, \frac{\mathbf{u}_2 + \mathbf{v}_2}{2} \right\rangle = \frac{1}{2}(\mathbf{u} + \mathbf{v})$

Initial point: $(\mathbf{u}_1, \mathbf{u}_2)$

Terminal point: $\frac{1}{2}(\mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2)$

$\text{w} = \left\langle \frac{\mathbf{u}_1 + \mathbf{v}_1}{2} - \mathbf{u}_1, \frac{\mathbf{u}_2 + \mathbf{v}_2}{2} - \mathbf{u}_2 \right\rangle$

$= \left\langle \frac{\mathbf{v}_1 - \mathbf{u}_1}{2}, \frac{\mathbf{v}_2 - \mathbf{u}_2}{2} \right\rangle = \frac{1}{2}(\mathbf{v} - \mathbf{u})$

8. Let $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \cdot \mathbf{w} = 0$.

Then, $\mathbf{u} \cdot (c\mathbf{v} + d\mathbf{w}) = \mathbf{u} \cdot c\mathbf{v} + \mathbf{u} \cdot d\mathbf{w}$

$= c\mathbf{u} \cdot \mathbf{v} + d\mathbf{u} \cdot \mathbf{w}$

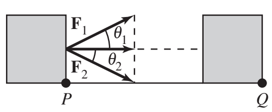
$= c(0) + d(0)$

$= 0.$

Thus for all scalars c and d, $\mathbf{u}$ is orthogonal to $c\mathbf{v} + d\mathbf{w}$.

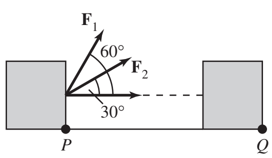
9. $W = (\cos\theta)\|F\|\,\|\overrightarrow{PQ}\|$ and $\|F_1\| = \|F_2\|$

(a)



If $\theta_1 = -\theta_2$ then the work is the same since $\cos(-\theta) = \cos\theta$.

(b)



If $\theta_1 = 60°$ then $W_1 = \frac{1}{2}\|F_1\|\,\|\overrightarrow{PQ}\|$

If $\theta_2 = 30°$ then $W_2 = \frac{\sqrt{3}}{2}\|F_2\|\,\|\overrightarrow{PQ}\|$

$W_2 = \sqrt{3}\,W_1$

The amount of work done by F_2 is $\sqrt{3}$ times as great as the amount of work done by F_1.

10. (a)

θ	$100 \sin\theta$	$100 \cos\theta$
0.5°	0.8727	99.9962
1.0°	1.7452	99.9848
1.5°	2.6177	99.9657
2.0°	3.4899	99.9391
2.5°	4.3619	99.9048
3.0°	5.2336	99.8630

(b) *No*, the airplane's speed does *not* equal the sum of the vertical and horizontal components of its velocity. To find speed:

speed $= \sqrt{(\|\mathbf{v}\|\sin\theta)^2 + (\|\mathbf{v}\|\cos\theta)^2}$

(c) (i) speed $= \sqrt{5.235^2 + 149.909^2} \approx 150$ miles per hour

(ii) speed $= \sqrt{10.463^2 + 149.634^2} \approx 150$ miles per hour

Chapter 6 Practice Test

For Exercises 1 and 2, use the Law of Sines to find the remaining sides and angles of the triangle.

1. $A = 40°,\ B = 12°,\ b = 100$

2. $C = 150°,\ a = 5,\ c = 20$

3. Find the area of the triangle: $a = 3,\ b = 6,\ C = 130°$.

4. Determine the number of solutions to the triangle: $a = 10,\ b = 35,\ A = 22.5°$.

For Exercises 5 and 6, use the Law of Cosines to find the remaining sides and angles of the triangle.

5. $a = 49,\ b = 53,\ c = 38$

6. $C = 29°,\ a = 100,\ b = 300$

7. Use Heron's Formula to find the area of the triangle: $a = 4.1,\ b = 6.8,\ c = 5.5$.

8. A ship travels 40 miles due east, then adjusts its course $12°$ southward. After traveling 70 miles in that direction, how far is the ship from its point of departure?

9. $\mathbf{w} = 4\mathbf{u} - 7\mathbf{v}$ where $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$. Find $\mathbf{w}$.

10. Find a unit vector in the direction of $\mathbf{v} = 5\mathbf{i} - 3\mathbf{j}$.

11. Find the dot product and the angle between $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.

12. $\mathbf{v}$ is a vector of magnitude 4 making an angle of $30°$ with the positive x-axis. Find $\mathbf{v}$ in component form.

13. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$ given $\mathbf{u} = \langle 3, -1 \rangle$ and $\mathbf{v} = \langle -2, 4 \rangle$.

14. Give the trigonometric form of $z = 5 - 5i$.

15. Give the standard form of $z = 6(\cos 225° + i \sin 225°)$.

16. Multiply $[7(\cos 23° + i \sin 23°)][4(\cos 7° + i \sin 7°)]$.

17. Divide $\dfrac{9\left(\cos \dfrac{5\pi}{4} + i \sin \dfrac{5\pi}{4}\right)}{3(\cos \pi + i \sin \pi)}$.

18. Find $(2 + 2i)^8$.

19. Find the cube roots of $8\left(\cos \dfrac{\pi}{3} + i \sin \dfrac{\pi}{3}\right)$.

20. Find all the solutions to $x^4 + i = 0$.